How Does a Market Perish?

Lessons from the Japanese Floaters Market

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Abstract

By investigating the recent demise of the 15-year Japanese floating rate bond (JF) market, we investigate the role of arbitragers in establishing market efficiency. If the market is supported by the massive presence of arbitragers, their synchronized and collective actions polish the market more effectively by eliminating mispricing quickly and sufficiently during tranquil market conditions. However, such a feature itself may dismantle the market during a liquidity driven crisis because they are all vulnerable to funding liquidity shocks. Such asymmetric role of arbitragers is highlighted by empirically investigating the term structure of liquidity implied by JFs.

Keywords: Arbitrage; Funding liquidity; Liquidity discount rate; Subprime crisis

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1 Introduction

$\cdots$ *The causes of extinction are usually multiple and linked to another in complicated synergy. But the precondition to extinction can be put in a single word: rarity.* $\cdots$

– D. Quammen, “The Song of the Dodo”

There has been a surge in academic literature on “liquidity risk” in the wake of the subprime turmoil in 2008. Arguably, existing literature classifies “liquidity” into three different categories: market liquidity, funding liquidity, and accounting liquidity. The market liquidity refers to the ease with which a particular asset is traded. Funding liquidity refers to the ease of external funding. Finally, accounting liquidity or balance sheet liquidity refers to the erosion of a financial institution’s capital on its balance sheet stirred by a setback in the asset prices it holds.

Brunnermeier and Pedersen (2009) shed new light on the interrelationship between funding liquidity and market liquidity. They propose that an initial loss to a financial institution near margin constraints or risk limits creates a funding problem, induces the institution to reduce its position, which itself moves the price against it and leads to further losses. This, in turn, increases market volatility and prompts a higher margin, which further tightens the institution’s funding constraint. Thus, all these downward pressures on liquidity feed each other in a vicious circle, which is called “liquidity spiral.” The balance sheet liquidity risk can be added to the liquidity spiral as an additional component. As the losses mount, systemically important financial institutions such as commercial banks, which are subject to tighter risk management including capital adequacy requirements, deleverage their balance sheets by offloading heavily risk-weighted assets, which, in turn, intensifies the downward move of the asset prices. Therefore, all types of risks intensify each other, which creates a vicious interlink spiral of risk.

Among the aforementioned liquidity types, market liquidity has drawn most attention in finance literature. Kyle (1985) suggests three essential criteria of market liquidity: tightness, depth, and resiliency.[1] Recently, Persaud (2001) adds diversity as an additional component: the degree of diversity among market participants in their market views and desired trades. However, market diversity should be, strictly speaking, counted as a pathogenesis while Kyle's components are symptoms given the fact that the lack of diversity among market participants is highly likely to widen the market breadth, make the market thin, and lessen market resiliency.

Based on the aforementioned advances in literature on liquidity risk, we empirically investigate 15-year Japanese floating rate notes (JFs, hereafter). JFs are a part of the Japanese Government Bond (JGB) market, which is second only to the U.S. Treasury market in size. The JF market is embedded with a number of intrinsic peculiarities that make it an ideal laboratory for investigating liquidity risk.

First, because JFs are government bonds, they are free from credit risk. Japan's sovereign debt risk is negligible and JGBs tend to be firmer under global financial market turmoils on the back of safety-haven demand.

Second, coupon payments for JFs are varying over time as their name implies, but they are almost perfectly replicable by fixed-coupon paying JGBs as shown later. Simply put, a JF is tantamount to a derivative security of regular JGBs and its fundamental value can be computed solely by the no arbitrage condition. As such, we can effectively single out market risk components so that any deviation in market price from the fundamental value is attributed to an imbalance between demand and supply, i.e., liquidity risk. Thus, we can directly focus on *market resiliency* rather than *market tightness* or *market depth*, which have been investigated in depth in the existing literature.[2]

[1]Tightness (or breadth) refers to the breadth of the bid-ask spread, which indicates the cost of a reversal of position at short notice for a non-extreme amount. Market depth is the size of an order that may be immediately executed without slippage of best limit order prices. Finally, market resiliency refers to the speed with which prices revert to their equilibrium level after a large shock in the transaction flow.

[2]The on-the-run phenomenon, which refers to the stylized fact that most recently issued bonds are more expensive and liquid than previously issued ones, has been the most popular evidence on the "resiliency" facet of liquidity. Some of these studies estimate the liquidity premia after adjusting for differentials in coupons and durations between on-the-runs and off-the-runs. Among many others, see Amihud and Mendelson (1991), Kamara (1994), Goldreich, Hanke and Nath (2005), and Pasquariello and Vega (2009) for evidence in U.S. Treasuries and Mason (1987) and Boudoukh and Whitelaw (1991, 1993) for that in JGBs. In contrast,

Third, because we can compute such illiquidity-driven mispricings of different JFs across time, we can retrieve a market-wide factor(s) that governs the time-series and cross-sectional variations of liquidity premia, and are able to construct a term structure of liquidity discount yields (LDYs), similar to a term structure of interest rates. To the best of our knowledge, this is the first attempt to investigate a term structure of LDYs.

Fourth, and most importantly, the JF market is an ideal place for gauging the "diversity" facet of liquidity proposed by Persaud (2001). From the outset of this market, a serious concern about the thinness of the investor universe and the lack of diversity was brought up since its payoff structure is atypical. JFs are Constant Maturity Treasuries (CMTs); the tenor of a JF's reference yield that determines its coupon rate does not match its payment interval. Due to the complexity of the payoff structure and the unfamiliar risk profile, the JF market has difficulty in broadening the spectrum of investors, especially among domestic real money managers, who do not rely on leverage in their funding. Instead, the JF market was successful in attracting foreign hedge funds, especially fixed-income arbitrage funds because JFs deliver ample opportunities for relative value (RV) trades against regular JGBs, and also against Constant Maturity Swaps. Since the RV traders are relatively homogeneous in timing the execution of trades, the lack of diversity among them is inherently prevalent in this market, which makes it vulnerable to a liquidity shock. The Japanese government recognized such shortcomings, albeit too late. Its Ministry of Finance (2009) stated

> $\cdots$ *prices of 15-year floating-rate bonds also plummeted. It is said that this was because overseas funds also held a certain amount of 15-year floating-rate bonds, and they also sold these when they unwound their trades* $\cdots$.

BIS has also noticed the massive presence of foreign RV funds in this market. In its Quarterly Review (2009), it stated

Longstaff (2004) compares the prices of U.S. Treasuries with those of U.S. agency bonds, which are guaranteed by the Treasury, thereby eliminating the market risk and credit risk altogether. He reports an enormous amount of liquidity premium in Treasury bonds, which can be more than fifteen percent of the value of some Treasuries. However, none of these studies explored either the term structure of liquidity or a relationship between market resiliency and funding liquidity.

> $\cdots$ *these bonds had come to be heavily held by (foreign) hedge funds speculating on a convergence between the market price and the higher theoretical price* $\cdots$ *The widening of the gap* $\cdots$ *is thought to reflect the recent shrinkage and deleveraging of hedge fund positions.*

Fifth, fixed-income arbitrage funds implement high-leverage strategies, where the leverage ratio typically ranges from five to ten times the capital input. As a consequence, these funds are fatally vulnerable to funding liquidity risk. Thus, this market is ideal for exploring evidence on the linkage between market liquidity risk and funding liquidity risk. Domestic investors are mostly real money managers, who are less vulnerable to an eruption of funding liquidity. It is foreign hedge funds who are susceptible to illiquidity in the funding market. Since most foreign hedge funds are funded on a dollar basis, this indicates that market illiquidity in the JF market should be more sensitive to the dollar funding liquidity rather than the yen funding liquidity, if market illiquidity is affected by funding illiquidity.

Finally, the Japanese Ministry of Finance put an end to the issuance of JFs in 2008 after observing the massive dislocation of this market. Thus, this market provides a rare opportunity for analyzing how a security market falls victim to a liquidity crisis and collpases.

We investigate how the JF market was dismantled by a liquidity shock by estimating the liquidity discount rate (LDR), a state variable that dictates the term structure of liquidity premia in yields from the cross-sectional dispersion in JF prices over time. To do this, we develop a discrete-time regime-shifting Vasicek model for the term structure of liquidity, which is similar to Bansal and Zhou (2002).[3]

We estimate our term structure model of liquidity by adopting a Bayesian Markov Chain Monte Carlo (MCMC) technique, by which we can estimate a latent factor, the structural parameters including the market price of risk, and the regimes.

The major findings are as follows. First, the Bayesian model comparison based on the log marginal likelihood delivers statistically significant evidence on the regime-shifting behavior

[3]However, the exact description of a stochastic process is slightly different between the two.

of LDR. The shift from the tranquil regime to the crisis regime occurred in October 2007, two months after BNP Paribas halted withdrawals from three investment funds under its management because it could not "fairly" value its holdings.

Second, the long-term mean of LDR during the crisis regime is as high as 7% per annum, whereas its value is trivial during the tranquil regime. In addition, its volatility during the crisis regime is three times higher than that during the tranquil regime.

Third, the market price of liquidity risk in both regimes is statistically insignificant. Under the model specified, this indicates that the market failed to price in such risk *a priori*, mainly because the probability that such a dramatic change in regimes occurs is perceived to be trivial.

Fourth, the estimated transition probability matrix demonstrates that the regimes are extremely persistent. The probability that the tranquil regime transits to the crisis regime is as low as one percent, whereas the probability that the crisis regime turns into the tranquil regime is almost zero.

Finally, our changepoint estimation results show that the eruption of funding liquidity crisis preceded that of the market liquidity crisis. In addition, the Granger causality test stipulates that market liquidity risk is not associated with funding liquidity risk during the tranquil regime. It is only in the crisis regime that funding liquidity becomes intertwined with market liquidity. More importantly, it is the dollar funding liquidity, and not the yen funding liquidity, that amplified market liquidity risk in the JF market.

The rest of the paper is organized as follows. Section 2 introduces the JF market along with its properties and delivers a valuation scheme of JFs based on the no arbitrage principle and shows its accuracy using a simulation. In Section 3, we explore a term structure of LDYs to explain the gap between JF market prices and their corresponding fair values. The Bayesian MCMC method is also introduced to estimate the model. Section 4 presents the estimation results of the theoretical model and investigates the relationship between JFs' funding liquidity and market liquidity. Section 5 discusses the implication of our analysis and concludes. Proofs and technical details are collected in appendix.

2 Introduction to the JF

JFs belong to Japanese Government Bonds (JGBs), which the Japanese government issues for financing purpose. Since the issuance of JF1 (the higher the post-fix figure, the more recent the issue) in 1983, JFs had been issued upto JF7 before the Japanese Ministry of Finance decided to stop issuing them in 1985. Their issuance resumed in 2000, and since then, five JFs have been, on average, issued per year with JF48 as the latest issue. The major features of JFs are as follows:

- Time to maturity at issuance: 15 years; the maturity date is the maturity month's 20th day (or next business day if the 20th falls on a holiday)
- Frequency of coupon payments: Semi-annual
- Annualized coupon rate: Max [basis rate + α, 0]
- Basis rate: Compounded yield of the average accepted bid of the 10-year JGB auction held six months before the ex-coupon date
- α: Floater spread called "quoted par margin"; it is determined in the auction of JFs and remains constant over its life
- Minimum amount of denomination: 100,000 yen

The most important variable in determining the price of a JF is α. As mentioned above, a bearer of a JF receives a semi-annual coupon payment of the 10-year JGB yield plus α. Because the typical shape of the yield curve is upward sloping, α is negative to ensure that JF price is at its par value at issuance date. Figure 1 illustrates how the cash flow stream of a JF with the time to redemption T (≤ 15) is reset over time.

In Figure 1, $y(t, 10)$ refers to the accepted average bid rate of a 10-year JGB auctioned at time t and M is the face value of a JF. $[x]^+ = \max[x, 0]$. Here, only the first coupon rate, $\frac{y(t_{i-1},10)+\alpha}{2}$ is known at time $t = t_i$, and the remaining coupon rates are random variables.

Let us discuss how α is supposed to be determined. As mentioned above, the price of a floater becomes a par value at each ex-coupon day if the time to maturity of the basis rate equals the frequency of the floater's coupon payment. For example, if the basis rate of a JF were the 6-month Japanese Treasury bill rate, its price with $\alpha = 0$ would converge to its par value at its issuance and each ex-coupon date. However, the JF's basis rate is the 10-year JGB par yield, which is greater than the 6-month Japanese Treasury bill rate in a generic upward sloping JGB yield curve. As such, without α, JF would pay too high coupon payments relative to its discount rates, which requires a premium in the price of the JF over par. To put the JF's price back to par at issuance, the market adds a negative α.[4]

In addition, Figure 1 shows that JF has a series of floorlets to prevent negative coupon payments. In case that an *ex post* 10-year JGB yield drops below $-\alpha$, the floorlet will be in force to turn the coupon payment to zero. To see this, we can decompose the coupon payment at time t_i into

$$\frac{[y(t_{i-1}, 10) + \alpha]^+}{2} = \frac{y(t_{i-1}, 10) + \alpha}{2} + \frac{\max[-\alpha - y(t_{i-1}, 10), 0]}{2}, \tag{2.1}$$

where $y(t_{i-1}, 10)$ is the 10-year JGB yield observed at time t_{i-1} and will be paid at time t_i. The second term in equation (2.1) is a payoff of a floorlet with exercise price $-\alpha$ and expiry t_i.

Table 1 documents the maturity date, α, and amount issued for each JF. It shows that both the frequency and the amount of issuance have declined after JF43, due to a plunge in demand for floaters driven by the global financial turmoil.

[4] By the same token, α would be positive if the JGB yield curve is downward sloping. In that sense, α is similar to a CMS (Constant Maturity Swap) spread (with negative sign) in a CMS contract.

The composition of the market participants in this market is noteworthy. As mentioned above, domestic real money managers and foreign hedge funds are the two major players in the JF market. Domestic real money managers include banks, investment trusts, and pension funds, which are regulated on their leverage. In contrast, most hedge funds present in the JF market are fixed-income arbitrage funds, which have expertise in relative value trades. Foreign investment banks' proprietary trading desks also participated in the market, and their investment strategies are akin to those of hedge funds. These arbitragers were known to account for a big portion of the JF market.[5] This seemingly innocuous market composition sustained a precarious equilibrium before the outbreak of the crisis.

The abovementioned payoff structure of JF sounds complicated but its valuation and corresponding hedge schemes are quite straightforward since its cash flows are almost perfectly replicable by fixed-coupon paying JGBs as shown below. Simply put, JFs are equivalent to derivative securities of fixed-coupon paying JGBs. In addition, its cash flows are isomorphic to those of CMS in which the basis rate is the 10-year swap rate as opposed to the 10-year JGB yield. Using an asset swap, JF can be exchanged with CMS. This "attainability" was a primary factor which made the market teemed with so many arbitragers.

2.1 Arbitrage pricing model for JFs

We first define the following notations:

$$\begin{aligned}
V(t) &= \text{value of a JF at time } t \\
r(t) &= \text{instantaneous interest rate at time } t \\
D(t,\tau) &= \text{time } t \text{ discount factor that discounts 1 yen at time } t+\tau \\
y(t_i) &= \text{semi-annually compounded par yield of a 10-year JGB } (= y(t_i, 10)) \\
N &= \text{number of coupon payments till the expiry of the JF.}
\end{aligned}$$

[5] See Bank of Japan (2008) and BIS Quarterly Review (June 2009). "Special Report on the Japanese Floating Rate Notes" of the Royal Bank of Scotland (2007) investigates the investor composition in the JF market in detail.

The fair value of a JF is the present value of its future cash flow, which can be expressed as follows under the equivalent martingale measure (EMM) Q:

$$\begin{aligned} V(t) &= E_t^Q\left[\sum_{i=1}^{N} e^{-\int_t^{t_i} r(s)ds}\frac{\max[y(t_{i-1})+\alpha,0]}{2}M + e^{-\int_t^{t_N} r(s)ds}M\right] && (2.2)\\ &= \sum_{i=1}^{N} E_t^Q\left[e^{-\int_t^{t_i} r(s)ds}\frac{\max[y(t_{i-1})+\alpha,0]}{2}\right]M + E_t^Q\left[e^{-\int_t^{t_N} r(s)ds}\right]M && (2.3)\\ &= \sum_{i=1}^{N} D(t,t_i-t)E_t^{Q_{D(t,t_i-t)}}\left[\frac{\max[y(t_{i-1})+\alpha,0]}{2}\right]M + D(t,t_N-t)M. && (2.4)\end{aligned}$$

In equation (2.4), we change the underlying probability measure from Q to $Q_{D(t,t_i-t)}$, a forward risk-neutral measure with respect to $D(t,t_i-t)$ ($\forall\ i = 1,2,\cdots,N$) by a numeraire change. Such a change also reflects an adjustment for time mismatch, which is discussed below.

In addition, the *ex-post* 10-year par yield at time t_i satisfies

$$M = \sum_{j=1}^{20} D(t_i,0.5j)\frac{y(t_i)}{2}M + D(t_i,10)M. \tag{2.5}$$

Then, the 10-year par yield can be expressed as a function of discount factors at time t_i:

$$y(t_i) = \frac{2(1-D(t_i,10))}{\sum_{j=1}^{20} D(t_i,0.5j)}. \tag{2.6}$$

In equation (2.3), $E_t^Q\left[e^{-\int_t^{t_i} r(s)ds}\frac{\max[y(t_{i-1})+\alpha,0]}{2}\right]M$ is similar to the value of a European call option on the 10-year JGB spot yield with $-\alpha$ as a strike yield. While interest rate options on the U.S. Treasury yields are listed on the Chicago Board Options Exchange, such instruments do not exist in Japan. In addition, it is not a standard European call, but a European call with deferred payment. The value of $y(t_{i-1})$ is realized at t_{i-1}, but it will not be paid until t_i. However, we can still value JFs by no-arbitrage restrictions alone since its payoff belongs to the attainable set of basis assets including fixed-coupon paying JGBs and swaptions as shown below.

In equation (2.4), $D(t, t_i - t)$, a discount factor, is the price of a zero-coupon bond with remaining time-to-maturity $t_i - t$. In Japan, the STRIPS market does exist. However, these are not actively traded and so we can observe only the quotes delivered by dealers. As an alternative, we can estimate the discount factors by interpolating the JGB yield curve. We use a cubic spline scheme based on Vasicek and Fong (1982) for all outstanding fixed-coupon paying JGB issues at each date.[6]

Plugging the discount factors obtained from the splining into equation (2.4) yields

$$V(t) = \sum_{i=1}^{N} D(t, t_i - t; \xi(t)) E_t^{Q_{D(t,t_i-t)}} \left[\frac{\max[y(t_{i-1}) + \alpha, 0]}{2} \right] M + D(t, t_N - t; \xi(t)) M, \quad (2.7)$$

where $\xi(t)$ represents the splining parameters. Then, the remaining task is to compute

$$E_t^{Q_{D(t,t_i-t)}} \left[\frac{\max[y(t_{i-1}) + \alpha, 0]}{2} \right] \quad \forall \;\; i = 1, 2, \cdots, N. \quad (2.8)$$

Using equation (2.1), we can rewrite equation (2.8) as

$$E_t^{Q_{D(t,t_i-t)}} \left[\frac{\max[y(t_{i-1}) + \alpha, 0]}{2} \right] = \frac{E_t^{Q_{D(t,t_i-t)}}[y(t_{i-1})] + \alpha}{2} + \frac{E_t^{Q_{D(t,t_i-t)}}[\max\{-\alpha - y(t_{i-1}), 0\}]}{2}. \quad (2.9)$$

In equation (2.9), the first term is determined by the forward 10-year par yield at time t_{i-1}, that is, the expected par yield under the forward measure. The second term is the expected payoff of a floorlet on that 10-year par yield with a strike yield of $-\alpha$ at its expiry. One issue in computing equation (2.9) is the mismatch in the timing of observation and payment of the coupon. $y(t_{i-1})$ is determined at time t_{i-1}, six months before the coupon payment is made. We first solve a case without time mismatch and then add an incremental value

[6] We also estimate the separate spot curves from on-the-run issues and from off-the-run issues and redo our analysis with these two yield curves to see whether our main results are robust to spreads between on-the-run and off-the-run issues, another popular measure of liquidity spreads. We find that the main results are qualitatively similar regardless of which yield curve we adopt. This result implies that our liquidity premium is not a trivial transformation of the on/off-the-run spreads and captures a separate liquidity component not reflected in the on/off-the-run spreads.

arising from the time mismatch by using a numeraire change.

In computing equation (2.9), under the assumption of no time mismatch, the first term is determined entirely by the forward yield curve. The second term, the expected payoff of the floorlet at its expiry, requires additional information, that is, the transitional distribution of the interest rate. There are two alternative arbitrage-free valuation schemes. First, we can use the no-arbitrage term structure model such as the Hull-White model and the Heath-Jarrow-Morton model. We assume a specific stochastic process of the short rate or the forward rate and retrieve the implied time-varying path of the key parameter(s) by calibrating the model to the observed yield curve. One drawback is that price is sensitive to the assumed stochastic process, which determines a sequence of interest rate distributions. Second, we can use bond options to directly retrieve the volatility path and thus the distribution of the interest rate. That is, we expand the basis assets and accordingly the attainable set. If the entire volatility skew is available, we do not even need to specify the stochastic process of the interest rate and can directly infer the transitional density of the future interest rate implied by the bond prices and the bond option prices.[7] We use the second scheme since it is less subject to potential errors arising from a misspecified stochastic process.[8]

Because the pricing procedure is highly technical, its details are described in Appendix. The resulting final pricing equation for JF is as follows:

$$V(t) \approx \sum_{i=1}^{N} D(t, t_i - t; \xi(t)) \left[\frac{\alpha + q(t_{i-1})}{2} + \frac{\zeta}{2} + \vartheta_{t,t_{i-1}} \right] M + D(t, t_N - t; \xi(t)) M, \qquad (2.10)$$

where

[7] In practice, the matrix of volatilities implied by options with different expiries and different moneyness is called the "volatility surface." In this paper, we use the interpolated volatility surface from swaption volatilities by using a kernel regression.

[8] However, the JGB bond option prices are, unfortunately, not available for the entire sample period. As an alternative, we utilize swaption prices and adjust for a difference in interest rate volatilities implied by bond options and swaptions based on volatility data from January 2006 to December 2006. The ratio between volatilities implied by the bond option prices and those implied by the swaption prices might be unstable over time. However, such misspecification errors are negligible since the total value of all floorlets (or floor) itself accounts for less than 0.1% of JF price. We also compute JF prices by using the Hull-White model based on historical volatilities. The resulting JF prices are almost indistinguishable from those from the second valuation scheme.

$$
\begin{aligned}
q(t_{i-1}) &= f_t(t_{i-1}) + \frac{\gamma(P(t_{i-1}, f_t(t_{i-1})), f_t(t_{i-1}))\sigma^2(t, f_t(t_{i-1}))}{2\delta(P(t_{i-1}, f_t(t_{i-1})), f_t(t_{i-1}))}, \\
\zeta &= \left[-(\alpha + q(t_{i-1}))\Phi(-z) + \sigma(t, f_t(t_{i-1}))\sqrt{t_{i-1} - t}\,\phi(z)\right], \\
z &= \frac{\alpha + q(t_{i-1})}{\sigma(t, f(t, t_{i-1}))\sqrt{t_{i-1} - t}}, \\
\vartheta_{t,t_{i-1}} &\approx -\frac{\rho\left(f_t(t_{i-1}), g_t(t_{i-1}, 0.5)\right)}{2(1 + g_t(t_{i-1}, 0.5)/2)}\sigma(g_t(t_{i-1}, 0.5))\sqrt{t_{i-1} - t} \\
&\quad \times\Phi(z)\left[\sigma(t, f_t(t_{i-1}))\sqrt{t_{i-1} - t}\left\{1 - \frac{\phi(z)}{\Phi(z)}\left(\frac{\phi(z)}{\Phi(z)} + z\right)\right\}\right. \\
&\quad \left. + \left(q(t_{i-1}) + \alpha + \frac{\phi(z)}{\Phi(z)}\sigma(t, f_t(t_{i-1}))\sqrt{t_{i-1} - t}\right)^2 (1 - \Phi(z))\right].
\end{aligned}
$$

Here $f_t(t_{i-1})$ is the forward 10-year yield for time t_{i-1}, as of time t; $q(t_{i-1})$ is the expected 10-year yield at time t_{i-1}; $g_t(t_{i-1}, 0.5)$ is the 6-month forward rate with forward time at t_{i-1}; and $\sigma(t, f_t(t_{i-1}))$ is the volatility of the forward rate. $\phi(\cdot)$ and $\Phi(\cdot)$ denote the normal probability density function and the normal cumulative distribution function, respectively. The approximation comes from two sources. First, the expected future yield under the forward measure is approximated by the forward yield combined with convexity adjustment. Note that the expected future zero-coupon bond price under the forward measure is the forward zero price, and the expected future yield of a coupon-paying bond under the forward measure is different from its forward yield, albeit slightly. To account for the difference, we use convexity adjustment. Second, the second moment of the spread between the future yield and the forward yield is approximated by the variance of the forward yield. In the next subsection, we show that such approximation errors are trivial enough to be ignored.

2.2 Simulation

Using a simulation, we investigate the accuracy of our pricing model. As discussed above, our valuation scheme, based on an arbitrage approach, attempts to minimize ad-hoc assumptions required in an equilibrium model so that any remaining errors may be driven by approximation errors. To gauge the magnitude of such errors, we consider a single-factor affine model with the following stochastic process of the instantaneous interest rate:

$$dr_t = \beta(\upsilon - r_t)dt + \epsilon dz_t, \quad \text{under } Q. \tag{2.11}$$

Using the above equation, we simulate a time series of the short rate. Then, we compute the 10-year par yield at each coupon reset date, t_i, of a given JF using the following formula:

$$\begin{aligned} y(t_i) &= \frac{2\left(1 - A(10)e^{-B(10)r_{t_i}}\right)}{\sum_{j=1}^{20} A(0.5j)e^{-B(0.5j)r_{t_i}}}, \\ \text{where} \quad A(\tau) &= exp\left((B(\tau)-\tau)\frac{\left(\beta^2\upsilon - \frac{\epsilon^2}{2}\right)}{\beta^2} - \frac{\epsilon^2 B(\tau)^2}{4\beta}\right) \\ B(\tau) &= \frac{\left(1-e^{-\beta\tau}\right)}{\beta}. \end{aligned}$$

Note that $A(\tau)e^{-B(\tau)r_{t_i}}$ is a zero-coupon bond price or equivalently a discount factor at t_i for 1 yen at time $t_i + \tau$. The simulated value of $y(t_i)$ is discounted by the discount factor using (2.3). We compute the fair value of a JF by simulating 100,000 time series of the short rate.

In the simulation, we adopt the following values of the structural parameters:

$$\beta = 0.1, \quad \upsilon = 0.025, \quad \epsilon = 0.009.$$

The resulting performance of our valuation model is tabulated in Table 2. For diverse combinations of β and υ, our model can approximate very accurately the true price of a JF over a spectrum of time to maturity from 1 year to 15 years. Its approximation error ranges from -.02 to .03, which is less than .04%. Therefore, we conclude that our approximation scheme is very accurate.

2.3 Properties of JFs

Generic bonds such as zeros or fixed-coupon paying bonds are financial instruments by which investors either speculate or hedge on the direction of the interest rate. If the interest rate is expected to make a downward move, buy them and vice versa. In addition, they tend to extend the duration of their bond portfolios under a bond-bullish view and shrink it under a bond-bearish view under the assumption of a parallel shift in the yield curve.

In addition, market participants also conduct so-called "curve trades." A representative curve trade is a spread or slope trade. For example, investors put on curve steepeners (flatteners) such as buying (shorting) 10-year bonds and shorting (buying) 20-year bonds if their view is that the long-end yield curve will steepen (flatten). The investment weight of slope trading is typically PVBP (price value of a basis point) or duration weighted to protect their position against a parallel shift in the yield curve.[9] However, such curve trades are relatively expensive to implement in a cash bond market because of the spread between the repo and reverse-repo rate. As a consequence, such trades are more popular in a swap space.

The duration risk of a JF is limited. When the yield curve shifts up, the discount rates increase, which puts a downward pressure on the prices of fixed-rate bonds. In contrast, the floating-rate bond prices are much less sensitive since their expected coupon payments (the 10-year forward yields) also increase concomitantly. Therefore, a JF is well insulated from such directional risk and accordingly its PVBP is relatively small.

Then, what is the primary driving force behind JF price changes? To answer this question, consider a 15-year JF. When the yield curve is upward sloping, both its expected coupon and discount rate increase as the time to its payment increases. However, the higher expected coupon incomes outrun the higher discount rates in JFs. For example, the last coupon payment will occur 15 years forward. Its value is determined in 14.5 years, which is

[9] Another popular curve trade is a butterfly: buying (shorting) short- and long-end bonds combined with shorting (longing) intermediate bonds with the expectation of strengthening (weakening) of curvature.

the reset time for the last coupon. The expected value of the 10-year par yield in 14.5 years is, in turn, determined by the expected zero rates from 15 years to 24.5 years. In contrast, the longest time duration of the discount rate applied in computing the present value of JF's cash flows is 15 years. Therefore, a higher slope of the yield curve results in the higher price of the JF.

To highlight this relationship between the yield curve slope and JF price, we consider a Vasicek model adopted in the previous subsection. We directly compute theoretical fixed-rate bond prices and JF prices using the same parameter values in the simulation.

Figure 2 shows that an increase in the yield slope increases JF price, especially the longer maturity JF. JF fair values increase with a higher slope between the 5-year yield and the 20-year yield (20-year yield minus 5-year yield). Its locus is not linear but convex such that JF price increasingly increases with steepening.

Therefore, we can conclude that JF is equivalent to a curve steepener. If the market participant expects that the JGB yield curve will steepen, he or she can buy a JF. He or she may short it if his or her view is curve flattening. Thus, unlike a fixed-rate JGB, JF delivers an opportunity for investing in a curve slope as opposed to the vertical direction of the curve.

The fact that JFs are curve steepeners suggests that a variety of relative value trades surrounding JFs are feasible. Suppose that a JF is traded below its fair value. One may monetize such underpricing by buying the JF along with putting on a curve flattener. One may express the curve flattener by longing 20-year bonds and shorting 5-year bonds in the cash bond market. Alternatively, one may consider receiving 20-years and paying 5-years in the JPY swap market. Another alternative would be to pay in the CMS swap. If you exert curve flatteners in a swap space, you should bear the volatility of asset swap spreads since overall trades mimic the so-called "box" trades: trading the differential between a slope in the bond yield curve and a slope in the swap curve. Consequently, such trades should be counted as a combination of arbitrage and statistical arbitrage. Such ample opportunities of relative value trades are the major driver for attracting so many fixed-income arbitrage funds into the JF market, whose demand makes up for relatively weak demand from domestic real money managers.

3 Identification of Market Liquidity Factors

In this section, we build a theoretical model of liquidity discounts in JF prices and explore an econometric method to gauge its empirical goodness-of-fit. Prior to that, we first need to discuss whether the deviation of JF market prices from their fair values is driven primarily by liquidity differentials between JFs and fixed-coupon paying JGBs.

In Section 2.1, we derive the fair values of JFs using discount factors and forward 10-year par yields, both of which are retrieved from the splined JGB yield curves. The fair values of JFs, therefore, reflect the liquidity of the fixed-coupon paying JGBs. As such, these fair values are "fair" to the extent that the liquidity in the JF market is the same as that in the regular JGB market. As a result, the differential of JF market prices from their fair values can be counted as the liquidity differential between the two markets.

Figure 3 depicts the time-series evolution of market prices coupled with the fair values of JF 25 from January 20, 2004 to October 20, 2010 on a monthly basis. This period includes the global financial turmoil, the inception of which is generally counted as August 2007, when BNP Paribas announced that it has failed to fairly value the underlying assets in three funds under its management as a result of exposure to the subprime mortgage lendings. The three shaded areas in Figure 3 represent the BNP Paribas event along with the acquisition of Bear Stearns by JP Morgan, and the collapse of Lehman Brothers. The spread of market price from the fair value become wider in the negative direction around the announcement of BNP Paribas and reached a peak when Lehman Brothers went bankrupt.[10]

Among potential drivers behind the observed behavior of JF mispricings in Figure 3, the

[10]For brevity, we report the figure for JF25 only. The observed time-series behaviors of the other JFs are qualitatively similar.

prime suspects would be credit risk and tax treatments, as other studies suggest. However, given that both JFs and fixed-rate JGBs are issued by the Japanese government and they are under the same taxation scheme, those two candidates can be immediately ruled out.

The only thinkable driver is a demand–supply imbalance, which results from a structural difference in investor compositions between the JF market and the fixed-rate JGB market. The fixed-rate JGB market is well supported by domestic real money managers. More than 90% of outstanding JGBs are held by them. In addition, JGBs are counted as a safety haven and as such, foreign investors were piling into them during the global financial crisis despite the fact that Japan is the world's most indebted country. In contrast, the JF market was characterized by weak demand from domestic real money managers, which was complemented by demand from foreign hedge funds. As discussed in Section 2, despite its unique merit as an investment vehicle, JF is too unacquainted and complicated for relatively conservative domestic real money managers. In addition, the ample opportunities for relative value trades entice foreign arbitrage funds to the JF market. However, these funds are inherently vulnerable to external shocks. Therefore, during the tranquil period, the JF market is sustainable; demand for JFs as an investment vehicle is relatively high so much that it could put upward pressure on market prices beyond their fair values. However, such an equilibrium is fragile. During the global financial crisis, these foreign hedge funds withdrew from the JF market, due to their binding capital constraints. The sudden pullout of foreign hedge funds tore a hole in the demand for JFs, which crashed the market. As mentioned in the introduction, the Debt Management Report published by the Japanese Ministry of Finance clearly confirms this inveterate weakness in the JF market. Overall, the dramatic changes in JF prices relative to their fair values during the critical event periods shown in Figure 3 indicates that the mispricing of JFs reflects a liquidity black hole in the JF market.

3.1 Term structure of LDYs

In this section, we provide a framework for analyzing liquidity discounts in JF prices. We consider a theoretical model that can coherently explain not only the cross-sectional dispersion in liquidity discounts across different JFs but also the time-series variation in them. To do so, we hire a modelling framework from the existing term structure model of interest rates. Specifically, we introduce liquidity discount factors (LDFs), which is an incremental discount factor associated with market illiquidity in JFs. The "usual" (time) discount factor is the factor by which a future cash flow must be multiplied in order to obtain the current market value. Such conversion of future cash flows into present values is already made in the fair value computation of JFs, as in equation (2.7). However, given the differential between JF market prices and the fair values, we need to introduce an additional discount (or premium) factor that stems from relative market illiquidity (or liquidity) in the JF market. As such, LDF bridges a gap between the fair value and the observed market price. Specifically, we consider the following valuation equation:

$$
\begin{aligned}
V_k(t)^a &= E^P\left[\Psi^k_{t,t+1} M_{t,t+1} V^a_k(t+1) | \mathcal{F}_{ta}\right] && (3.1)\\
&= \sum_{i=1}^{N} E^P\left[\Psi^k_{t,t+i} M_{t,t+i} CF(t+i) | \mathcal{F}_{ta}\right]. && (3.2)
\end{aligned}
$$

In the above equations, $\mathcal{F}_{ta}$ is the P-augmented sigma algebra, that is, $F_{ta} = \sigma\left(\mathcal{F}_t\right) \bigcup \sigma\left(\mathcal{F}_{tL}\right)$ where $\mathcal{F}_t$ is the smallest sigma algebra generated by $M_{t,t+1}$ and $V_k(t+1)$, and $\mathcal{F}_{tL}$ is the smallest sigma algebra generated by Ψ^k_{t+1}. $M_{t,t+i}\left(= \prod_{j=1}^{i} M_{t+j-1,t+j}\right)$ is the generic stochastic discount factor. In contrast, $\Psi^k_{t,t+1}$ is defined such that

$$
\Psi^k_{t,t+1} = \begin{cases} 1 & \text{if } k \in \text{fixed-rate JGBs,} \\ M^L_{t,t+1} & \text{if } k \in \text{JFs} \end{cases}
$$

where $M^L_{t,t+1}$ is the stochastic *liquidity* discount factor, which further discounts or inflates the discounted future cash flows of JFs. Therefore, equation (3.1) indicates that the generic stochastic discount factor, M_{t+1}, is extended to $\Psi^k_{t,t+1} M_{t,t+1}$. In addition, obviously $\Psi^k_{t,t+i} = \prod_{j=1}^{i} \Psi^k_{t+j-1,t+j}$.

From here on, we express the one-period stochastic liquidity discount factor $M_{t,t+1}$ as M_{t+1} for brevity. Rewriting equation (3.1) yields

$$\begin{aligned} E_t^P\left(1 + r_k(t+1)\right) \;\; = \;\; & \frac{1}{E_t^P(\Psi^k_{t+1}) E_t^P(M_{t+1})} - \frac{1}{E_t^P(M_{t+1})} \mathrm{Cov}(M_{t+1}, r_k(t+1)) \\ & - \frac{1}{E_t^P(\Psi^k_{t+1}) E_t^P(M_{t+1})} \mathrm{Cov}\left(\Psi^k_{t+1}, M_{t+1} r_k(t+1)\right). \end{aligned} \tag{3.3}$$

Equation (3.3) indicates that the expected gross return on a regular JGB collapses to a standard one since $\Psi^k_{t+1} = 1$ for the fixed-rate JGB. As for a JF, equation (3.3) states that there are two adjustments. The first term is further discounted by $E_t^P(\Psi^k_{t+1}) = E_t^P\left(M^L_{t+1}\right)$. The third term is newly introduced and reflects that investors may require compensation for comovement between the return discounted by the generic stochastic discount factor and and the stochastic liquidity discount factor. In this paper, we assume that the comovement is zero, that is, $\mathrm{Cov}\left(\Psi^k_{t+1}, M_{t+1} r_k(t+1)\right) = 0$, for two reasons. First, remember that we adopt a no-arbitrage approach in extracting M_{t+1} from the JGBs rather than modeling it. In contrast, we will theoretically model M^L_{t+1}. Combining the empirically retrieved M_{t+1} and theoretically specified M^L_{t+1} is still feasible, but we need to make ad-hoc assumptions on their time-series relationship. Second, and more important, $M_{t+1} r_k(t+1)$ is a variable that determines the fundamental or fair value. Therefore, $\mathrm{Cov}\left(\Psi^k_{t+1}, M_{t+1} r_k(t+1)\right)$ reflects the comovement between the stochastic liquidity discount factor (a demand-supply imbalance) and the fair value of JF (primarily driven by the slope of the yield curve). It is hard to believe, *a priori*, that the fair value of the JF is strongly associated with its illiquidity, and the behavior of JF25's fair price and its mispricing in Figure 3 confirms no relationship between the two. That is, a change in the liquidity condition in the JF market is externally driven by a change in demand-supply imbalance, and not internally induced by a change in the fair value of the JFs. Given such a premise, Cov $\left(\Psi^k_{t+1}, M_{t+1} r_k(t+1)\right) = 0$ would be an innocuous assumption. Put differently, $\sigma\left(\mathcal{F}_t\right) \bigcap \sigma\left(\mathcal{F}_{tL}\right) = \emptyset$. Then, we can rewrite equation (3.3) as

$$V_k(t)^a = \sum_{i=1}^{N} E^P \left[\Psi^k_{t,t+i} | \mathcal{F}_{tL}\right] E^P \left[M_{t,t+i} CF(t+i) | \mathcal{F}_t\right]. \tag{3.4}$$

Therefore, we need to theoretically model $E^P \left[\Psi^k_{t,t+i} | \mathcal{F}_{tL}\right]$ separately from the fair value.

We formally define LDF such that

$$\text{LDF}(t, t_i - t) = E^P \left[M^L_{t,t_i} | \mathcal{F}_{tL}\right], \tag{3.5}$$

which refers to the liquidity-adjusted time t value of *a present value* of a yen at time t_i. LDF can be represented as a function of liquidity discount yield (LDY), $y_L(t, t_i - t)$, such that

$$\text{LDF}(t, t_i - t) = \exp\left[-(t_i - t) y_L(t, t_i - t)\right]. \tag{3.6}$$

Similar to the zero bond price, LDF at the maturity date should be equal to one since liquidity discount vanishes at the maturity date; that is,

$$\text{LDF}(t, 0) = 1 \ \ \forall \ \ t.$$

However, unlike the generic discount factor, LDF can exceed one before the maturity date since JFs can be more liquid than fixed-rate JGBs, albeit rarely, as evidenced in Figure 3. This means that $y_L(t, t_i - t)$ might be negative. As such, the properties of LDFs (LDYs) are exactly identical to those of *real* bond prices (spot yields).

Following the term structure literature, we consider a liquidity discount rate (LDR), l_t, which is equivalent to the short rate such that

$$\text{LDF}(t, t_i - t) = E^Q \left[\exp\left(-\int_{s=t}^{t_i} l_s ds\right) | \mathcal{F}_{tL}\right].$$

We need to specify l_t. We consider a single-factor model wherein l_t is an affine function of a single factor, x_t:

$$l_t = \delta_0 + \delta_1 x_t.$$

Without loss of generality, we set $\delta_0 = 0$ and $\delta_1 = 1$ (therefore, $l_t = x_t$) to avoid identification problems. A critical assumption required is the stochastic process of x_t. First, we consider the following AR(1) process in a discrete time framework, which is equivalent to the Ornstein-Uhlenbeck process in continuous time, which is explored in the Vasicek model:

$$x_{t+1} - x_t = \kappa(\mu - x_t) + \omega \, u_{t+1}, \tag{3.7}$$

where κ, μ, and ω are the speed of mean-reversion, long-term mean, and volatility, respectively. u_{t+1} is a conditionally standard normal variate given x_t. One desirable feature of this process is that the domain of x_t is a set of real numbers. As mentioned above, the fact that x_t could be sign switching is a feature required to conform to the behavior of LDFs.

We enrich the stochastic process of x_t by engrafting a regime-swtiching behavior on equation (3.7). One of the widely-known properties of liquidity risk is that it makes a quantum leap during the crisis whereas it is dormant in tranquil times.[11]

To accommodate such a property, we consider a two-state Markov regime shift process as in Hamilton (1989). The transitional probability matrix of a Markov chain that governs the evolution of the sequel regime $s_{t+1} = 1, 2$ given the current regime $s_t = 1, 2$ is

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}.$$

[11] A number of recent studies argue that liquidity risk is subject to a regime-shifting behavior. For example, Wu (2012) argues that liquidity risk is dormant in undisturbed market climates, but it turns into the most important risk factor once its value exceeds a certain threshold. After an eruption of liquidity risk, it does not follow a mean-reverting pattern, and feeds on itself, gathers momentum, and causes more severe market declines. Similarly, Bervas (2006) also suggests that liquidity risk should be decomposed into "normal" liquidity risk in tranquil periods and "extreme" liquidity risk which becomes activated during financial crises.

We can rewrite the process of latent factors when the economy is subject to regime shifts as:

$$x_{t+1} - x_t = \kappa_{s_{t+1}}(\mu_{s_{t+1}} - x_t) + \omega_{s_{t+1}} u_{t+1}, \tag{3.8}$$

where $\kappa_{s_{t+1}}$, $\mu_{s_{t+1}}$, and $\omega_{s_{t+1}}$ are now regime-dependent parameters.

The LDF, M^L_{t+1}, is specified as:

$$M^L_{t+1} = \exp\left[-l_t - \frac{1}{2}\left(\frac{\lambda_{s_{t+1}}}{\omega_{s_{t+1}}}\right)^2 - \frac{\lambda_{s_{t+1}}}{\omega_{s_{t+1}}} u_{t+1}\right] \tag{3.9}$$

where $\lambda_{s_{t+1}}$ is the market price of risk, which also depends on s_{t+1}.

Conjecturing the functional form of $\mathrm{LDF}_{s_t}(t, \tau)$ as

$$\mathrm{LDF}_{s_t}(t, \tau) = \exp\left[-A_{s_t}(\tau) - B_{s+t}(\tau) x_t\right], \tag{3.10}$$

the solution for LDY[12] is

$$y_{L,s_t}(t, \tau) = -\frac{\ln \mathrm{LDF}_{s_t}(t, \tau)}{\tau} = \frac{A_{s_t}(\tau)}{\tau} + \frac{B_{s_t}(\tau) x_t}{\tau},$$

where

$$\begin{bmatrix} B_1(\tau) \\ B_2(\tau) \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \begin{bmatrix} (1-\kappa_1)B_1(\tau-1)+1 \\ (1-\kappa_2)B_2(\tau-1)+1 \end{bmatrix},$$

$$\begin{bmatrix} A_1(\tau) \\ A_2(\tau) \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \begin{bmatrix} A_1(\tau-1) + (\kappa_1\mu_1 - \lambda_1)B_1(\tau-1) - \frac{1}{2}B_1(\tau-1)^2\omega_1^2 \\ A_2(\tau-1) + (\kappa_2\mu_2 - \lambda_2)B_2(\tau-1) - \frac{1}{2}B_2(\tau-1)^2\omega_2^2 \end{bmatrix}.$$

3.2 Estimation methodology

We now discuss how to estimate the structural model on LDF. Given that the underlying model requires estimation in a state space, we need to simultaneously estimate the underlying LDR and the structural parameters governing the term structure of LDYs. One may consider

[12] We impose the boundary condition $A_{s_t}(0) = B_{s_t}(0) = 0$ and a restriction $A_{s_t}(1) = 0$, $B_{s_t}(1) = 1$ for $s_t = 1, 2$.

a number of alternative econometric schemes such as Kalman filtering, Efficient Method of Moments (EMM), and indirect inference methods. Unfortunately, these methods are either infeasible or difficult to apply due to the irregularity of the likelihood surface. The irregularity or multi-modality of the likelihood function with respect to the parameters and factors is caused by the nature of our data, the structure of which is highly persistent, and the severe nonlinearity of the model. Further, a time series of a particular JF is not time homogeneous in its property due to time lapse. Simply put, today's JF is not the same as yesterday's in terms of time-to-maturity. As a result, the maximum likelihood estimation or EMM based on an optimization method is never reliable because the estimation results are extremely sensitive to the choice of the starting values in the optimization.

In existing empirical literature on term structure models, this problem can be avoided by constructing an artificial bond, the time-to-maturity of which is kept the same over the entire sample period by using a spline method. Our data set consists of JFs that do not span the entire spectrum of time-to-maturities and thus, such splining is not a viable option. As such, we have to directly deal with time-inhomogeneous JFs. There exists one feasible estimation method that can overcome and handle such non-stationarity in data and the irregular likelihood surface. It is a nonlinear space estimation scheme adopting an efficient Bayesian MCMC method, and our MCMC method is the tailored multiple-block Metropolis-Hastings (M-H) algorithm (Chib and Ergashev (2009)).

3.2.1 Nonlinear state-space representation

For posterior inference, we represent the resulting econometric model into a state space form. We begin with the measurement equation describing the relationship between the vectors of observed security prices and the model-implied prices:

$$\log V(t)^{\text{market price}} = \log V(t)^a + e(t),$$

where $V(t)^a$ is the vector of liquidity-adjusted theoretical price that is expressed in (3.4) and $e(t)$ is the vector of pricing errors. The pricing errors are assumed to be mutually independent and normally distributed, and their mean is zero and variance is $\sigma_{s_t}^2$ given s_t. Conditioned on the regimes, the transition equation is given by the first-order autoregressive factor process:

$$x_{t+1} - x_t = \kappa_{s_{t+1}}(\mu_{s_{t+1}} - x_t) + \omega_{s_{t+1}} u_{t+1}, \quad u_{t+1} \sim \text{i.i.d.} \mathcal{N}(0, 1).$$

The model parameters to be simulated are denoted by $\theta = \{\kappa_{s_t}, \mu_{s_t}, \omega_{s_t}, \lambda_{s_t}, \pi_{s_t,s_t}, \sigma_{s_t}^2\}_{s_t=1,2}$. We complete our econometric modeling by specifying the prior as follows:

$$\begin{aligned}
\kappa_{s_t} &\sim beta(19, 4), \\
\mu_2 &\sim \mathcal{N}(0, 0.25), \\
170 \times \omega_1 &\sim inverse\ gamma(40, 40), \\
80 \times \omega_2 &\sim inverse\ gamma(40, 40), \\
\lambda_{s_t} &\sim \mathcal{N}(0, 0.25), \\
\pi_{s_t,s_t} &\sim beta(54, 3), \\
10^4 \times \sigma_{s_t}^2 &\sim inverse\ gamma(40, 40) \text{ for } s_t = 1, 2.
\end{aligned}$$

The choice of the prior is based on the simulation-based method as suggested in Chib and Ergashev (2009). The prior distribution is set to generate a positive liquidity premium on average while allowing for a flexible shape of the term structure. To identify the factor and to avoid the label-switching problem, we impose the following identifying restrictions

$$|\kappa_{s_t}| < 1, \ 0 < \pi_{s_t,s_t} < 1, \ \mu_1 = 0, \ \omega_{s_t} > 0 \text{ for all } s_t.$$

Within regime 1, the unconditional mean of the factor process is zero, and it is possibly nonzero in regime 2. These restrictions are enforced through the prior.

3.2.2 Posterior distribution and MCMC sampling

Our goal is to efficiently simulate the posterior joint distribution of the parameters, time series of the factor, and regime. The efficiency of our MCMC sampler is measured by the inefficiency factor, which is computed as

$$1 + \frac{2 \times B}{B-1} \sum_{j=1}^{B} K(j/B)\hat{\rho}(j), \tag{3.11}$$

where $\hat{\rho}(j)$ is the sample autocorrelation at lag j calculated from the MCMC draws and $K(\cdot)$ is the Parzen kernel (Kim, Shephard, and Chib (1998)). A low inefficiency factor indicates an efficient sampling and convergence of the Markov chain (Chib (1996)).

Suppose that $y_t = \log P_t^{\text{market price}}$, $\mathbf{y} = \{y_t\}_{t=1}^{T}$ is the set of the observed security prices, $\mathbf{S} = \{s_t\}_{t=1}^{T}$, and $\mathbf{X} = \{x_t\}_{t=1}^{T}$. Then, the posterior to be simulated is given by

$$\pi(\theta, \mathbf{X}, \mathbf{S}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{X}, \mathbf{S},\theta)p(\mathbf{X}|\mathbf{S},\theta)p(\mathbf{S}|\theta)\pi(\theta),$$

where $p(\mathbf{y}|\mathbf{S}, \mathbf{X}, \theta)$ is the conditional joint density of $\mathbf{y}$, and it is obtained from the measurement equation:

$$p(\mathbf{y}|\mathbf{X}, \mathbf{S},\theta) = \prod_{t=1}^{T} p(y_t|\mathbf{X}, \mathbf{S},\theta),$$

where $p(y_t|\mathbf{X}, \mathbf{S},\theta) = \mathcal{N}(y_t|\log V_t^a, \sigma_{s_t}^2)$. $p(\mathbf{X}|\mathbf{S}, \theta)$ is factor density conditioned on the regimes and parameters as described in the regime-switching factor process. It is computed as

$$\begin{aligned} p(x_t|x_{t-1}, \mathbf{S}, \theta) &= \mathcal{N}(x_t|x_{t-1} + \kappa_{s_t}(\mu_{s_t} - x_{t-1}),\omega_{s_t}^2), \\ p(\mathbf{X}|\mathbf{S},\theta) &= \prod_{t=1}^{T} p(x_t|x_{t-1}, \mathbf{S}, \theta). \end{aligned}$$

$p(\mathbf{S}|\theta)$ is the regime density of the Markov-switching regimes conditioned on the transition probabilities. Let $n_{i,j}$ denote the counts of transitions from regime i to regime j during the sample period. Then, the joint density of the regimes is given by

$$p(\mathbf{S}|\theta) = p(\mathbf{S}|\pi_{1,1}, \pi_{2,2}) = \pi_{1,1}^{n_{1,1}} \times (1 - \pi_{1,1})^{n_{1,2}} \times (1 - \pi_{2,2})^{n_{2,1}} \times \pi_{2,2}^{n_{2,2}}.$$

Finally, $\pi(\theta)$ is the prior density of θ. We simulate the joint posterior distribution by an MCMC simulation method, in which each MCMC cycle consists of four stages.[13]

4 Empirical Analysis

4.1 Data

JF market prices are obtained from Bloomberg on a daily basis. We set a unit time interval at one month, that is, 1/12 year. Since JF maturities fall on the 20th day of their months of redemption, we use the monthly observations under the 20th of every month from the dataset.[14] At each point in time, we discard on-the-run observations in our cross-sectional data with the concern that they may be affected by idiosyncratic demand.[15] The resulting data period ranges from July, 2002 to October, 2010.

Table 3 documents the number of observations and descriptive statistics of liquidity discounts across JFs. It shows that the number of observations for each JF is not identical over time because of the sequential issuance of JFs. Consequently, the number of cross-sectional data points in each date increases with the issuance of a new JF. In addition, though not shown here, the shortest maturity available across the entire sample period is 4.7 years and thus the entire spectrum of time-to-maturities is not observable. Because of this problem, we could not adopt the popular scheme of splining the term structure curve of liquidity yield. As mentioned above, this is the critical reason we use the efficient Bayesian MCMC method in our estimation.

For all JFs, average liquidity discounts are negative, which indicates that JFs have been traded, on average, below their corresponding fair values. In addition, absolute values are larger for recently issued JFs, which implies that the discounts are a more evident feature in

[13] The details on algorithm of MCMC sampling are available upon request.

[14] When the 20th day falls on weekend or holiday, we take the most adjacent observation.

[15] The on-the-run issues are artificially boosted by the Ministry of Finance in Japan. We re-conduct our empirical analysis by including the on-the-run issues. The estimation results are almost identical. We regard a JF as an on-the-run issue as long as there is no subsequent JF.

the later part of the data period. One thing to notice is the dates when the price differentials reach their minimum values. While those dates on which maximums occur are rather dispersed over time, most dates on which minimums occur are centered on March 2008 and the fourth quarter of 2008, which coincide with the collapses of Bear Stearns and Lehman Brothers, respectively.

4.2 Estimation of the LDR

Before discussing the estimation results on the latent factor (LDR) and structural parameters, we first explore whether the introduction of regime-switching is justifiable, that is, whether it significantly contributes to enhancing the goodness-of-fit of the model. Table 4 reports the result for the Bayesian model comparison. It shows that the regime-switching model's log marginal likelihood value is much higher than that of the corresponding non-switching model, and that the posterior probability of the regime-switching model is one whereas that of the non-switching model is zero. Thus, we can conclude that the regime-shifting behavior of LDR is statistically well supported by the market prices of JFs.

Next, we investigate how much of the liquidity discounts of JFs can be explained by our term structure model of market liquidity. In Figure 4, the solid line illustrates the time-series of cross-sectional mean squared log differences between the market prices and their fair values. Similarly, the dotted line depicts those between the market prices and their liquidity-adjusted values implied by the regime-shifting model. Therefore, the differences between the two time series dynamics illustrate how much of the observed price discrepancies between the market prices and their fair values are explained by JFs' market illiquidity implied by the theoretical model. Figure 4 shows that the two time series are not noticeably distinguishable before late 2007; introducing illiquidity as an additional factor does not add much to explaining the observed mispricing of JFs before the outbreak of the subprime mortgage crisis. However, after late 2007, a substantial portion of JFs' mispricing is explained by the illiquidity in the JF market, as designated by our theoretical model. About 85% to

90% of the mispricing during the period can be explained by the term structure of the liquidity model.

Figure 5 exhibits the time-series evolution of the estimated LDR along with the estimated posterior probability of regime 2. Both LDR and the posterior probability of the second regime made a quantum jump in late 2007, and did not fall until 2010. Before late 2007, LDR has exhibited a sign-switching behavior, which indicates that JFs were often traded at a premium over their fair values. Then, LDR soared in late 2007, which means that the market began to discount JFs further due to their relative illiquidity, and from then on, LDR stayed high. Even when the impact of the financial crisis retreated and other financial markets normalized after 2009, JFs' liquidity fails to recover its pre-crisis level. Basically, the JF market collapsed and never resurrected.

Table 5 documents the statistical properties of the estimated parameters. The mean reversion speeds of regime 1 and regime 2, κ_{s_t}, are 0.40 and 0.39, respectively, which are almost identical. In contrast, the long-term means of regime 1 and regime 2, μ_{s_t}, are 0.00 and 0.07, respectively. The steady-state mean of LDR is essentially zero in regime 1, which indicates that JFs are fairly priced in the long run. In contrast, that of LDR is as high as 7% annually in regime 2, which is extraordinarily high given the fact that Japan's short rate is essentially zero. Therefore, the change in the long-term mean of LDR is the key driving source of the regime shift. The market price of liquidity risk (λ_{s_t}) is statistically insignificant in both regimes. In particular, it is somewhat surprising that its estimate is zero in regime 2. This indicates that the market participants fail to demand for risk premium on massive liquidity risk. The variance of log pricing errors of the model, $\sigma^2_{s_t}$, is indistinguishable between the two regimes. Both regimes look persistent as shown in the estimate of π_{s_t,s_t}. However, regime 1 is transient and non-recurrent while regime 2 is an absorbing state as the estimate of $\pi_{1,1}$ is less than one ($\pi_{1,1} = 0.99$), but that of $\pi_{2,2}$ is equal to one. That is, once LDR enters regime 2, it never exits out of it, which is coherent with the finding in Figure 5. Therefore, in fact, regime 2 is not only a high illiquidity state but also a state of market collapse (liquidity blackhole)!

Figure 6 illustrates the term structure of LDYs, which is retrieved from the estimated LDR and parameters. It is evident that the term structure exhibits drastic stepwise changes at the inception of the subprime crisis and stayed high afterwards. The term structure also shows an upward sloping curve at the inception of the crisis, then changed its shape to a downward sloping curve at the pinnacle of the crisis such as Bear Stearns and Lehman scandals.

4.3 Market liquidity and funding liquidity

In this section, we investigate the relation between market liquidity and funding liquidity based on the time series of LDR estimated in the previous subsection. This relation has drawn a lot of attention recently since the pioneering theoretical works of Brunnermeier and Pedersen (2009) and Brunnermeier (2009). However, the empirical analysis is still at an early stage.[16] Since we have a clean measure of market liquidity, our analysis provides an ideal laboratory for investigating this relation directly.

4.3.1 Funding liquidity measures and structural breaks

The LDR estimated in the previous subsection serves as a measure of market liquidity in the JF market. Using this measure, we explore the relation between market liquidity and funding liquidity. We consider two funding liquidity measures, Japanese yen TED and (US) TED spreads. The Japanese TED spread (JPTED) is a spread between the 3-month yen Libor rate and the 3-month Japanese Treasury bill rate. The 3-month yen Libor rate reflects the short-term yen funding cost of major domestic commercial banks. Thus, an increase in the JPTED spread is a sign that financial institutions' yen funding has become relatively more expensive indicating that the funding liquidity of yen has worsened. Given that JFs

[16] See Ben-David, Franzoni and Moussawi (2012) and Jylhä, Rinne and Suominen (2014). These studies directly investigate the relation between hedge funds' supply of liquidity and funding liquidity condition.

are primarily traded in Japan, the first natural candidate for funding liquidity is the JPTED spread.

The TED spread is similarly defined as a spread between the 3-month dollar Libor rate and the 3-month US Treasury bill rate. As such, it reflects the funding liquidity condition in dollar. The reason that the TED spread may be a relevant funding liquidity measure in the JF market is twofold. First, the subprime crisis, though originating from the US, became a global crisis, and so it is reasonable to consider funding conditions outside Japan. Second, and more importantly, foreign hedge funds are important investors in the JF market and they face capital constraints from their head offices abroad; as such, they are seriously affected by the liquidity conditions in their funding currencies.

As mentioned before, the Japanese Ministry of Finance stated that the debacle in the JF market during the crisis was primarily driven by the overseas funds' sudden liquidation of positions in this market. Should its statement be correct, we may find a stronger relationship between the TED spread and market liquidity in the JF market than the one between the JPTED spread and market liquidity. It empirically provides a clue to our premise that arbitragers taper off market liquidity during crises and may even completely dismantle the market if their portion of the market is sufficiently large.

Figure 7 illustrates market liquidity (LDR) and the aforementioned two funding liquidity measures. The fluctuations in the two funding liquidity measures and LDR have much in common.[17] In particular, the TED and JPTED spreads and LDR seem to have undergone a one-time structural break around 2007. It is not surprising that the funding conditions in the global financial market as well as the local market deteriorated drastically during the subprime crisis. An interesting feature is that the drying up of funding liquidity did not evolve gradually over time but suddenly made its presence.

We first estimate the exact timing of structural changes in their dynamics. We formally examine the changepoints based on Chib's approach by estimating the following first-order

[17] We sometimes omit the term "spread" in JPTED spread and TED spread for expressional conciseness.

autoregressive process with an unknown changepoint:

$$LM_{t+1} - c_{q_{t+1}} = g_{q_{t+1}}(LM_t - c_{q_t}) + v_{q_{t+1}}\iota_{t+1}, \tag{4.1}$$

where ι_{t+1} ~i.i.d.$\mathcal{N}(0,1)$, and LM=TED, JPTED, or LDR. q_t is assumed to follow a two-state Markov process in which the regime shift is permanent.[18] All parameters (c_{q_t}, $g_{q_{t+1}}$, and $v_{q_{t+1}}$) are subject to break at an unknown changepoint. By using the multi-move method suggested by Chib (1998), we are able to generate the posterior distribution of the changepoints as well as the parameters. Figure 8 plots the posterior probabilities of the changepoint of each liquidity measure over time, and Table 6 presents the posterior mean and 95% credibility intervals of the parameters and changepoints. The changepoints of the TED, JPTED, and LDR dynamics are estimated in June, March, and October 2007, respectively. For all liquidity measures, the driving nature of the break is found to be the upward shifts in the mean and conditional variance (i.e., $c_{q_t=2} > c_{q_t=1}$ and $v_{q_t=2} > v_{q_t=1}$) as shown in Table 6. Two months after the increase in the mean and volatility of the TED and JPTED spreads, LDR shifted upward subsequently and dramatically. The changepoint estimate of the JPTED spread is March 2007, which is earlier than that of the TED spread, June 2007. However, substantial portions of their confidence intervals overlap in Table 6. In fact, such difference in changepoints is statistically insignificant, and simply refects that the changepoint in the TED spread is more accurately estimated. More importantly, while the changepoints of the TED and JPTED spreads are not substantially different, they clearly antedated that of LDR. The 95% credibility interval of the changepoint of LDR is far beyond the estimated changepoint of TED and JPTED. Thus, we can conclude that the drying up of funding liquidity antecedes the worsening of market liquidity whereas the worsening of yen funding liquidity takes place almost at the same time as that of dollar funding liquidity.

[18] We assume symmetric priors across the states, and the states are identified by the information contained in the data, rather than the prior. Specifically, for $q_t = 1, 2$, $\mu_{q_t} \sim N(0, 1)$, $g_{q_t} \sim N(0.7, 0.1)$, and $\omega_{q_t} \sim IG(51, 0.5)$. The transition probability is given as $\Pr[q_{t+1} = 1|q_t = 1] \sim beta(9, 10)$. Restricting $\Pr[q_{t+1} = 2|q_t = 2] = 1$, the first state is non-recurrent. For the detailed posterior sampling procedure, refer to Chib (1998).

4.3.2 Relations between market liquidity and funding liquidity

To clarify the dynamic interactions among the three liquidity measures in a simple and tractable way, we estimate the following two equations:

$$\mathrm{LDR}_t = \text{constant} + \phi_1 \mathrm{LDR}_{t-1} + \phi_2 \mathrm{TED}_{t-1} + \phi_3 \mathrm{JPTED}_{t-1} + e_{Lt}, \tag{4.2}$$

$$\mathrm{JPTED}_t = \text{constant} + \rho_1 \mathrm{LDR}_{t-1} + \rho_2 \mathrm{TED}_{t-1} + \rho_3 \mathrm{JPTED}_{t-1} + e_{Jt}, \tag{4.3}$$

where the errors are normally, identically, and independently distributed[19]. By estimating the first equation we can investigate how market liquidity is affected by global and local funding liquidity. We are also interested in the feedback from market liquidity to local funding liquidity, which is the reason the second equation is estimated. We do not analyze the feedback from market liquidity to dollar funding liquidity because the JF market is not large enough to make any impact on dollar funding. We split the samples into pre- and post-crisis periods. The sub-sample for the pre-crisis period is until February 2007, and the sub-sample for the post-crisis starts from October 2007, as indicated by our changepoint estimation results in Table 6.

Table 7 summarizes the estimation results. Panel A indicates that neither TED nor JPTED influenced LDR during the pre-crisis period. In contrast, during the post-crisis period, an increase in the TED spread significantly increased LDR because the posterior probability of a positive ϕ_2 is 91%, whereas JPTED decreased LDR, albeit insignificantly.[20] These results have important implications for the relationship between funding liquidity and market liquidity. A worsening of either global funding liquidity or local funding liquidity does not systematically affect market liquidity of JFs in a tranquil market. However, once crisis erupts, it is dollar funding, and not yen funding that matters in the JF market. This is direct

[19] The intercept and the slope coefficients are assumed to follow standard normal *a priori*, and the prior for the error variance is set to be diffuse.

[20] The correlation coefficients between the liquidity measures are substantially higher during the crisis regime. Over the tranquil regime, corr(LDR,TED) = 0.13, corr(LDR,JPTED) = 0.16, and corr(TED,JPTED) = 0.13. After the crisis, corr(LDR,TED) = 0.55, corr(LDR,JPTED) = 0.42, and corr(TED,JPTED) = 0.79.

evidence that the collapse of the JF market is linked to the exodus of foreign arbitragers (RV traders). A rise in dollar funding coupled with binding capital constraints triggered by a plunge in JF prices cornered foreign arbitragers into liquidating their existing positions in JFs, which further pressured the prices of JFs below their fundamental values. In addition, note that most foreign investors are arbitragers, and this is the reason why LDR is sensitive to dollar funding during the crisis. Therefore, we can conclude that the collapse of the JF market was triggered by the emigration of "arbitragers." This finding supports the argument of the Japanese Ministry of Finance and also our premise that when a large portion of a particular market is comprised of arbitragers, such a market is more heavily hit during a crisis because arbitragers are more vulnerable to funding liquidity.[21]

Second, Panel B demonstrate that the TED spread has a remarkable impact on the JPTED spread after the crisis. It can be said that global funding conditions play a more critical role in determining local funding conditions during the crisis. The impact of LDR on JPTED is not significant. This finding does not necessarily imply the absence of a feedback impact of market liquidity on funding liquidity: it simply means that JPTED may not reflect funding liquidity relevant to the JF market.

5 Implications and Conclusion

While most asset markets suffering from the subprime credit crunch normalized, the JF market failed to come back. Specifically, the issuance of JFs was called off due to the lack of sufficient demand after the outbreak of the crisis. What do we learn from the demise of the JF market?

[21] This finding is consistent with Jylhä, Rinne and Suominen (2014), who document that although hedge funds typically supply liquidity, they demand liquidity during crises. However, their study focuses on hedge funds' demand for or supply of liquidity, and not its impact on price.

5.1 Rarity, diversity, and arbitragers

The obsolescence of JFs delivers important lessons regarding the viability of a market. Note that the JF market had nothing to do with the U.S. subprime mortgage but still fell victim to it. What makes this market so vulnerable to the seemingly unrelated crisis and eventually makes it extinct? This is because of the rarity and lack of diversity in market participants. Rarity comes from the fact that the participation of domestic real money managers in this market was limited and domestic hedge funds specializing in Japanese fixed-income securities were basically absent. That is, the JF market failed to attract a large enough size of "resident" population whose major habitat is the Japanese market. The lack of demand from domestic investors was filled with demand from foreign arbitrage funds, which are intrinsically migratory in their investment.

A related issue is the lack of diversity among investors. Hedge funds specializing in fixed-income arbitrage are extremely similar in their key strategies. They seize a trade opportunity when the gap between the market price of a JF and its replicating portfolio widens above a pre-specified level. They unwind their positions either when the spread contracts to a certain level (profit realization) or when it widens further to a pre-specified level (loss cut). Therefore, their entry into and exit from trades are very similar albeit their exact profit realization levels and loss cut levels are slightly different. Simply put, their investment strategies are uni-directional and devoid of diversity. As a result, regardless of the number of hedge funds participating in the JF market, they act as a huddled mass.

Under a tranquil market condition, such synchronized collective actions among arbitragers have the benefits of polishing the market more effectively by eliminating mispricings quickly and sufficiently. However, when the market is embroiled in turmoil, such singular actions of arbitragers become detrimental to the market. It amplifies mispricings because of the liquidity spiral suggested by Brunnermeier and Pedersen (2009). Homogeneity among funds tend to strengthen during a crisis because of funding liquidity. Funds' investment strategies, like other relative value strategies, tend to be highly leveraged.[22] As such, when JF prices plummet and the spread suffers from large dislocations during the crisis, these funds are requested to inject more capital due to an erosion in the collateral value of their positions and margin hikes. Highly leveraged funds that cannot roll over their short-term liabilities cannot help unwinding their long positions on JFs and short positions on replicating portfolios. These funds tend to liquidate their positions almost at the same time: these are "crowded trades" as mentioned by Brunnermeier (2009). Moreover, fire sales of JFs triggered by initial victims exacerbate the price decline, which adds pressure on the next funds in the queue that survived the initial price shock. In summary, the inherent homogeneity of arbitrage funds contributes to making the market more efficient under normal conditions, but it can dismantle the market during times of crises. So the arbitragers are a double-edged sword.

5.2 Implications for security design

A natural question is why the JF market suffered from rarity and lack of diversity in the first place? The following statement from the Japanese Ministry of Finance (2010) may shed light on this:

> *As for the floating-rate bonds (issuance has been suspended), it is necessary for the issuing authorities to examine the potential market needs or the possibility of product redesign, taking into account that the existing products are not internationally common and there are some views suggesting that they have complicated and difficult risk profiles.*

The answer is that JF itself was ill-designed. On one hand, its cash flow streams and related risk profiles were too complicated for relatively "conservative" Japanese investors. Most Japanese institutional investors are long driven on the level of yields and have lim-

[22] Most of these funds are dollar funded. As such, the liquidity spiral also suggests funding liquidity as another source of international contagion in a financial crisis.

ited experience of trading on "yield slopes." Furthermore, its payoff structure riddled with complicated components such as floorlets and quoted par margins along with non-classical duration risk significantly attenuated demand from domestic real money managers. In addition, this kind of product is not available in other countries; simply put, JF is endemic to Japan, which alienates foreign real money managers. On the other hand, the fact that its payoff is almost replicable attracts foreign relative value funds, which are similar in their underlying strategies. To summarize, this product was not marketable enough to procure sufficient demand from real money managers, which play an important role of breakwater against external shocks. Instead, the nature of its design lures foreign hedge funds that are vulnerable to external shocks, thereby further amplifying the spiral effects that arise during crises. These features made the market less viable and more fragile and eventually led to its failure.

Allen and Gale (1988) show theoretically that new securities should be designed such that in every state, all payoffs are allocated to the security held by the group that values it most. That is, optimal security is extreme security that maximizes the clientele effect. Such a conclusion is correct in a static world where the issuing entity issues securities only once. In a dynamic world where the issuer repeats the issuance of securities over time, they should also pay attention to the sustainability of the security market. If they split the payoff too finely to attract a particular group of *homogeneous* clients, the issuer can maximize the total market value of future payoff but at the same time introduce the risk of market extermination arising from the homogeneity of market participants.

5.3 Summary

Unlike other markets, the JF market has failed to be resurrected after the crisis because many quant-based fixed-income arbitrage funds were washed away by the financial tsunami and could not return safely. The number of arbitrage funds who survived was not large enough to collectively "polish" the market. Note that these funds are not local hedge funds who specialize in Japanese fixed-income markets and, for these funds, there were ample opportunities for scavenging elsewhere in the dislocated global fixed-income market. After their fund size shrank and they could capture trade opportunities with better risk-return profiles elsewhere, they did not need to come back to the JF market. In addition, the surviving market participants recognized that the other survivors were in a similar situation and thus their collective and synchronized move was not restorable to the pre-crisis level. This is the reason the JF market still suffers from the enormous amount of mispricing.

We summarize a critical lesson from the demise of the JF market. Minimizing rarity and maximizing diversity among market participants is essential for market sustainability. To accomplish this, the product should be well designed to attract enough demand from investors who are less leveraged with long investment horizon and less homogeneous. Otherwise, a clientele effect may make its presence in a way that is detrimental to the sustainability of the market. Poor product design may bring in unwelcome investors who may disperse external shocks and precipitate a crisis.

Appendix: Solving Equation (2.8)

A Without Time Mismatch

Instead of equation (2.9), we compute the following corresponding value in absence of a time mismatch:

$$E_t^{Q_{D(t,t_i-t)}}\left[\frac{\max[y(t_i)+\alpha,0]}{2}\right] = \frac{E_t^{Q_{D(t,t_i-t)}}[y(t_i)]+\alpha}{2} + \frac{E_t^{Q_{D(t,t_i-t)}}[\max\{-\alpha-y(t_i),0\}]}{2}. \tag{A.1}$$

Unlike equation (2.9), the 10-year yield inside the bracket is $y(t_i)$, the timing of which coincides with the maturity date of a zero-coupon bond underlying the forward risk-neutral probability measure, $Q_{D(t,t_i-t)}$, in equation (A.1). We solve the two terms in equation (A.1) separately.

1st Term: $\frac{E_t^{Q_{D(t,t_i-t)}}[y(t_i)]+\alpha}{2}$

As shown in equation (2.5), the *ex-post* 10-year par yield at time t_i, $y(t_i)$ should satisfy the following equation:

$$M = \sum_{j=1}^{20} D(t_i, 0.5j)\frac{y(t_i)}{2}M + D(t_i,10)M \qquad (= P(t_i, y(t_i)))\,. \tag{A.2}$$

We first compute the forward 10-year yield, $f_t(t_i)$, which equates the forward price of the 10-year bond to a par value, where the forward price can be obtained by adding the expectation operator under the forward risk-neutral measure with respect to $Q_{D(t,t_i-t)}$:

$$\begin{aligned} M &= \sum_{j=1}^{20} E_t^{Q_{D(t,t_i-t)}}\left[D(t_i,0.5j)\frac{f_t(t_i)}{2}\right]M + E_t^{Q_{D(t,t_i-t)}}[D(t_i,10)]M \\ &= \left[\sum_{j=1}^{20} E_t^{Q_{D(t,t_i-t)}}[D(t_i,0.5j)]\right]\frac{f_t(t_i)}{2}M + E_t^{Q_{D(t,t_i-t)}}[D(t_i,10)]M \end{aligned}$$

$$= \left[\sum_{j=1}^{20} F_t(t_i, 0.5j)\right] \frac{f_t(t_i)}{2} M + F_t(t_i, 10) M,$$

where $F_t(t_i, 0.5j)$ ($\forall\ j = 1, 2, \cdots, 20$) is a $(t_i - t)$ year into $0.5j$ year forward zero-coupon price determined at t, which is

$$F_t(t_i, 0.5j) = \frac{D(t, t_i + 0.5j - t)}{D(t, t_i - t)},$$

which is driven by a no-arbitrage condition. Therefore, the desired forward 10-year yield can be expressed as

$$\begin{aligned} f_t(t_i) &= \frac{2\left(1 - F_t(t_i, 10)\right)}{\sum_{j=1}^{20} F_t(t_i, 0.5j)} \\ &= \frac{2\left(D(t, t_i - t) - D(t, t_i + 10 - t)\right)}{\sum_{j=1}^{20} D(t, t_i + 0.5j - t)}. \end{aligned}$$

However, $E_t^{Q_{D(t,t_i-t)}}[y(t_i)] \neq f_t(t_i)$. To compute $E_t^{Q_{D(t,t_i-t)}}[y(t_i)]$, we take a Taylor expansion on equation (A.2) with respect to $y(t_i)$ around the expansion point $f_t(t_i)$:

$$\begin{aligned} M &= P(t_i, y(t_i)) \\ &= P(t_i, f_t(t_i)) + P'(t_i, f_t(t_i))(y(t_i) - f_t(t_i)) + \frac{1}{2} P''(t_i, f_t(t_i))(y(t_i) - f_t(t_i))^2 + O_3. \end{aligned}$$

We take the expectation on both sides under $Q_{D(t,t_i-t)}$:

$$\begin{aligned} M &= E_t^{Q_{D(t,t_i-t)}}[P(t_i, y(t_i))] \\ &\approx M + P'(t_i, f_t(t_i)) \left[E_t^{Q_D(t,t_i-t)}(y(t_i)) - f_t(t_i)\right] + \frac{1}{2} P''(t_i, f_t(t_i)) E_t^{Q_{D(t,t_i-t)}} \left[y(t_i) - f_t(t_i)\right]^2. \end{aligned}$$

Finally, we can express $E_t^{Q_D(t,t_i-t)}(y(t_i))$ as

$$q(t_i) \stackrel{let}{=} E_t^{Q_D(t,t_i-t)}(y(t_i)) \tag{A.3}$$

$$
\begin{aligned}
&\approx \quad f_t(t_i) - \frac{P''(t_i, f_t(t_i)) E_t^{Q_{D(t,t_i-t)}} \left[y(t_i) - f_t(t_i)\right]^2}{2P'(t_i, f_t(t_i))} \\
&= \quad f_t(t_i) - \frac{\frac{P''(t_i, f_t(t_i))}{P(t_i, f_t(t_i))} E_t^{Q_{D(t,t_i-t)}} \left[y(t_i) - f_t(t_i)\right]^2}{2\frac{P'(t_i, f_t(t_i))}{P(t_i, f_t(t_i))}} \qquad \text{(A.4)} \\
&\approx \quad f_t(t_i) + \frac{\gamma(P(t_i, f_t(t_i)), f_t(t_i)) \sigma^2(t_i, f_t(t_i))}{2\delta(P(t_i, f_t(t_i)), f_t(t_i))}, \qquad \text{(A.5)}
\end{aligned}
$$

where $\delta\left(P(t_i, f_t(t_i)), f_t(t_i)\right)$ and $\gamma\left(P(t_i, f_t(t_i)), f_t(t_i)\right)$ are (modified) duration and convexity of the 10-year par JGB at forward yield $f_t(t_i)$, respectively. Further from equation (A.4) to (A.5), we use

$$
\sigma^2(t_i, f_t(t_i)) \approx E_t^{Q_{D(t,t_i-t)}} \left[y(t_i) - f_t(t_i)\right]^2,
$$

where $\sigma(t_i, f_t(t_i))$ is the volatility of the 10-year JGB yield at time t_i. There are two ways of computing this value. First, we use an estimate of historical volatility. However, this is the volatility defined under the physical measure, which approximates $E_t^{Q_{D(t,t_i-t)}} \left[y(t_i) - f_t(t_i)\right]^2$ poorly. A better alternative is to use the normalized volatility implied by either an option on the yield or a JGB bond option, but those derivatives do not trade over a full range of required expiries. As a result, we use a swaption volatility matching the expiry precisely. To adjust the difference between the par JGB yield volatility and the swaption volatility, we estimate the historical ratio of the two Black-Scholes volatilities, one implied by the 10-year JGB futures and the other implied by the swaption over the period from January 2006 to December 2006. The estimated ratio is 0.7, which, we assume, remains constant. Then, we convert it into a normalized volatility and thus obtain

$$
\sigma(t_i, f_t(t_i)) \approx 0.7 \sigma_{BS}^{swaption}(t, t_i) \sqrt{t_i - t} f_t(t_i), \qquad \text{(A.6)}
$$

where $\sigma_{BS}^{swaption}(t, t_i)$ is a Black-Scholes swaption volatility with expiry $t_i - t$. Equation (A.5) indicates that the expected 10-year par yield under the risk-neutral measure can be approximated by the forward yield with an adjustment. The adjustment term, $\frac{\gamma(P(t_i, f_t(t_i)), f_t(t_i)) \sigma^2(t_i, f_t(t_i))}{2\delta(P(t_i, f_t(t_i)), f_t(t_i))}$, aka "convexity adjustment" is positive and increases with higher volatility.

2nd Term: $\frac{E_t^{Q_{D(t,t_i-t)}}[\max\{-\alpha-y(t_i),0\}]}{2}$

The second term, the expected payoff of a floorlet, requires a cumulative distribution function. As discussed above, we assume a normal distribution with a given swaption-implied normal volatility of the 10-year JGB yield in equation (A.6):[23]

$$y(t_i) \sim N\left(q(t_i), \sigma^2(t_i, f_t(t_i))(t_i - t)\right) \quad \text{under } Q_{D(t_i-t)}. \tag{A.7}$$

Lemma 1: The first and second moments of $E[\max(x,0)]$ are

$$\begin{aligned} E[\max(x,0)] &= \mu\Phi(z) + \phi(z)\sigma, \\ \text{Var}[\max(x,0)] &= \Phi(z)\left[\sigma^2\left\{1 - \frac{\phi(z)}{\Phi(z)}\left(\frac{\phi(z)}{\Phi(z)} + z\right)\right\} + \left(\mu + \frac{\phi(z)}{\Phi(z)}\sigma\right)^2(1 - \Phi(z))\right], \end{aligned}$$

where

$$\begin{aligned} x &\sim N(\mu, \sigma^2), \\ z &= \frac{\mu}{\sigma}, \\ \phi(\cdot) &= \text{normal probability density function}, \\ \Phi(\cdot) &= \text{normal cumulative distribution function}. \end{aligned}$$

Proof: The first moment is

$$\begin{aligned} E[\max(x,0)] &= E(x|x>0)\cdot \text{Prob}(x>0) \\ &= \left[\mu + \frac{\phi\left(-\frac{\mu}{\sigma}\right)}{1-\Phi\left(-\frac{\mu}{\sigma}\right)}\sigma\right]\left[1 - \Phi\left(-\frac{\mu}{\sigma}\right)\right] \\ &= \mu\left[1 - \Phi\left(-\frac{\mu}{\sigma}\right)\right] + \phi\left(-\frac{\mu}{\sigma}\right)\sigma \end{aligned}$$

[23] Note that we do not necessarily assume a normal distribution on the time t 10-year JGB par yield. We use an actual swaption volatility surface determined in the market. A market convention is to quote swaption volatilities by normal volatilities rather than Black-Scholes volatilities. That is, whatever their actual pricing scheme is, the resulting volatilities are quoted as "implied normal volatilities." This does not mean that market participants use the normal model in their valuation of swaption premia.

$$= \mu\Phi\left(\frac{\mu}{\sigma}\right) + \phi\left(\frac{\mu}{\sigma}\right)\sigma,$$

where we use $\phi(z) = \phi(-z)$ and $1 - \Phi(z) = \Phi(-z)$.

The second moment is

$$\begin{aligned}
\mathrm{Var}[\max(x,0)] &= E\left(x^2|x>0\right)\cdot \mathrm{Prob}(x>0) - E(x|x>0)^2 \cdot \mathrm{Prob}(x>0)^2 \\
&= \left[\mathrm{Var}(x|x>0) + E(x|x>0)^2\right]\cdot \mathrm{Prob}(x>0) - E(x|x>0)^2\cdot \mathrm{Prob}(x>0)^2 \\
&= \mathrm{Prob}(x>0)\left[\mathrm{Var}(x|x>0) + E(x|x>0)^2(1-\mathrm{Prob}(x>0))\right] \\
&= \Phi(z)\left[\mathrm{Var}(x|x>0) + \left(\mu + \frac{\phi(z)}{\Phi(z)}\sigma\right)^2 (1-\Phi(z))\right] \\
&= \Phi(z)\left[\sigma^2\left\{1 - \frac{\phi(z)}{\Phi(z)}\left(\frac{\phi(z)}{\Phi(z)} + z\right)\right\} + \left(\mu + \frac{\phi(z)}{\Phi(z)}\sigma\right)^2(1-\Phi(z))\right]. \qquad Q.E.D.
\end{aligned}$$

Using this lemma, we can compute the second term

$$\begin{aligned}
&\frac{E_t^{Q_{D(t,t_i-t)}}[\max\{-\alpha - y(t_i), 0\}]}{2} \\
&= \frac{1}{2}\left[-(\alpha + q(t_i))\Phi\left(-\frac{\alpha + q(t_i)}{\sigma(t_i, f_t(t_i))\sqrt{t_i - t}}\right)\right. \\
&\quad \left. +\sigma(t, f_t(t_i))\sqrt{t_i - t}\,\phi\left(-\frac{\alpha + q(t_i)}{\sigma(t, f_t(t_i))\sqrt{t_i - t}}\right)\right]. \qquad \text{(A.8)}
\end{aligned}$$

B Time Adjustment

Now, equation (A.1) can be computed from equations (A.5) and (A.8), but for convenience, we change the time index from t_i to t_{i-1} in equation (2.9) such that

$$\begin{aligned}
&E_t^{Q_{D(t,t_{i-1}-t)}}\left[\frac{\max[y(t_{i-1}) + \alpha, 0]}{2}\right] = \\
&\underbrace{\frac{E_t^{Q_{D(t,t_{i-1}-t)}}[y(t_{i-1})] + \alpha}{2}}_{A} + \underbrace{\frac{E_t^{Q_{D(t,t_{i-1}-t)}}[\max\{-\alpha - y(t_{i-1}), 0\}]}{2}}_{B}, \qquad \text{(B.1)}
\end{aligned}$$

where

$$
\begin{aligned}
A &= \frac{q(t_{i-1})+\alpha}{2}, \\
B &= \frac{1}{2}\left[-(\alpha+q(t_{i-1}))\Phi(-z)+\sigma(t_{i-1},f_t(t_{i-1}))\sqrt{t_{i-1}-t}\;\phi(z)\right], \\
z &= \frac{\alpha+q(t_{i-1})}{\sigma(t_{i-1},f_t(t_{i-1}))\sqrt{t_{i-1}-t}}, \\
q(t_{i-1}) &= f_t(t_{i-1})+\frac{\gamma(P(t_{i-1},f_t(t_{i-1})),f_t(t_{i-1}))\sigma^2(t_{i-1},f_t(t_{i-1}))}{2\delta(P(t_{i-1},f_t(t_{i-1})),f_t(t_{i-1}))}.
\end{aligned}
$$

In equation (2.9), the underlying probability measure is $Q_{D(t,t_i-t)}$, not $Q_{D(t,t_{i-1}-t)}$ in equation (B.1). Thus, the final adjustment is a time adjustment by changing the numeraire underlying the forward probability measure from $Q_{D(t,t_{i-1}-t)}$ to $Q_{D(t,t_i-t)}$.

Following Harrison and Pliska (1981), the underlying numeraire change requires an adjustment, $\vartheta_{t,t_{i-1}}$, such that

$$
E_t^{Q_{D(t,t_i-t)}}\left[\frac{\max[y(t_{i-1})+\alpha,0]}{2}\right] = E_t^{Q_{D(t,t_{i-1}-t)}}\left[\frac{\max[y(t_{i-1})+\alpha,0]}{2}\right]+\vartheta_{t,t_{i-1}}, \quad \text{(B.2)}
$$

where

$$
\begin{aligned}
\vartheta_{t,t_{i-1}} &= \int_t^{t_{i-1}} \operatorname{cov}\left(dx(s),\frac{d\Lambda(s))}{\Lambda(s)}\right)ds, \\
x(s) &= \frac{\max[y(s)+\alpha,0]}{2}, \\
\Lambda(s) &= \frac{D(s,t_i-s)}{D(s,t_{i-1}-s)}.
\end{aligned}
$$

Unless we pre-specify the stochastic differential equation about state variables underlying the term structure of JGB yields, we cannot solve for $\vartheta_{t,t_{i-1}}$. However, we can still approximate its value under the assumption that correlation between the 6-month zero rate and the 10-year JGB yield is constant.

First, the numeraire ratio can be expressed as a function of the $(t_{i-1}-t)$ into 6-month forward rate, $g_t(t_{i-1}, 0.5)$:

$$\Lambda(t) = \frac{1}{(1 + g_t(t_{i-1}, 0.5)/2)^{0.5}}.$$

Therefore, from Ito's lemma, we can show that

$$\begin{aligned}
\sigma\left(\frac{d\Lambda(t)}{\Lambda(t)}\right) &= \frac{\partial\Lambda(t)/\partial g_t(t_{i-1}, 0.5)}{\Lambda(t)}\sigma(g_t(t_{i-1}, 0.5)) \\
&= -\frac{1}{2(1 + g_t(t_{i-1}, 0.5)/2)}\sigma(g_t(t_{i-1}, 0.5)).
\end{aligned}$$

Under the assumption that this value is constant, the sum of diffusion terms is

$$\int_t^{t_{i-1}} \sigma\left(\frac{d\Lambda(s)}{\Lambda(s)}\right) ds \approx -\frac{1}{2(1 + g_t(t_{i-1}, 0.5)/2)}\sigma(g_t(t_{i-1}, 0.5))\sqrt{t_{i-1} - t}. \tag{B.3}$$

In contrast, given the normal volatility of the 10-year JGB forward yield, the distribution of $x(t_{i-1})$ is

$$x(t_{i-1}) \sim N\left(q(t_{i-1}) + \alpha, \sigma^2(t_{i-1}, f_t(t_{i-1}))(t_{i-1} - t)\right).$$

Using Lemma 1 yields

$$\begin{aligned}
&\int_t^{t_{i-1}} \sigma(dx(s))ds \approx \\
&\quad \Phi(z)\left[\sigma^2(t_{i-1}, f_t(t_{i-1}))(t_{i-1} - t)\left\{1 - \frac{\phi(z)}{\Phi(z)}\left(\frac{\phi(z)}{\Phi(z)} + z\right)\right\}\right. \\
&\quad \left. + \left(q(t_{i-1}) + \alpha + \frac{\phi(z)}{\Phi(z)}\sigma(t_{i-1}, f_t(t_{i-1}))\sqrt{t_{i-1} - t}\right)^2 (1 - \Phi(z))\right],
\end{aligned}$$

which is a more precise approximation to the volatility of $x(t)$ than the one inferred from Ito's lemma. Note that the two volatilities are determined by the 10-year forward yield and 6-month zero rate, respectively. We assume that the historical correlation coefficient of the two rates, $\rho(df_t(t_{i-1}), dg_t(t_{i-1}, 0.5))$, is constant. Then, the resulting time adjustment can be written as follows:

$$\begin{aligned}
\vartheta_{t,t_{i-1}} &\approx \left[\int_t^{t_{i-1}} \sigma(dx(s))ds\right]\left[\int_t^{t_{i-1}} \sigma\left(\frac{d\Lambda(s)}{\Lambda(s)}\right)ds\right] \\
&= -\frac{\rho\left(df_t(t_{i-1}), dg_t(t_{i-1}, 0.5)\right)}{2(1+g_t(t_{i-1}, 0.5)/2)}\sigma(g_t(t_{i-1}, 0.5))\sqrt{t_{i-1}-t} \\
&\quad \times\Phi(z)\left[\sigma^2(t_{i-1}, f_t(t_{i-1}))(t_{i-1}-t)\left\{1-\frac{\phi(z)}{\Phi(z)}\left(\frac{\phi(z)}{\Phi(z)}+z\right)\right\}\right. \\
&\quad \left.+\left(q(t_{i-1})+\alpha+\frac{\phi(z)}{\Phi(z)}\sigma(t_{i-1}, f_t(t_{i-1}))\sqrt{t_{i-1}-t}\right)^2(1-\Phi(z))\right]. \qquad \text{(B.4)}
\end{aligned}$$

Since $\rho\left(df_t(t_{i-1}), dg_t(t_{i-1}, 0.5)\right)$ is positive, $\vartheta_{t,t_{i-1}}$ is negative.

Plugging equations (B.1) and (B.4) into (B.2) yields the value of $E_t^{Q_{D(t,t_i-t)}}\left[\frac{\max[y(t_{i-1})+\alpha,0]}{2}\right]$. In turn, plugging such a value for each coupon payment into equation (2.7) results in a desired valuation of the JF.

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Table 1: **Outstanding Issues of Japanese Floaters (JFs)**

	Maturity date	α	Amt issued		Maturity date	α	Amt issued
JF8	06-22-2015	-81	506.5	JF29	07-20-2019	-97	1,087.2
JF9	09-21-2015	-81	568.1	JF30	09-20-2019	-98	1,099.0
JF10	12-21-2015	-89	750.4	JF31	11-20-2019	-101	1,301.7
JF11	03-20-2016	-89	779.3	JF32	01-20-2020	-101	1,404.5
JF12	06-20-2016	-94	799.8	JF33	03-20-2020	-100	1,393.5
JF13	09-20-2016	-99	799.4	JF34	05-20-2020	-101	1,487.7
JF14	12-22-2016	-98	989.6	JF35	07-20-2020	-95	1,488.8
JF15	03-20-2017	-99	999.1	JF36	09-20-2020	-95	1,421.2
JF16	05-20-2017	-100	895.1	JF37	11-20-2020	-75	1,439.6
JF17	07-20-2017	-103	899.1	JF38	01-20-2021	-65	1,568.2
JF18	09-22-2017	-95	899.7	JF39	03-20-2021	-45	1,359.9
JF19	11-20-2017	-86	889.2	JF40	05-20-2021	-40	1,099.3
JF20	01-20-2018	-75	891.2	JF41	07-20-2021	-45	1,151.2
JF21	03-20-2018	-70	998.5	JF42	09-20-2021	-50	1,188.2
JF22	05-20-2018	-55	999.0	JF43	11-20-2021	-50	1,139.5
JF23	07-20-2018	-61	999.1	JF44	02-20-2022	-45	934.5
JF24	09-20-2018	-57	999.0	JF45	05-20-2022	-45	952.9
JF25	11-20-2018	-69	998.8	JF46	08-20-2022	-40	999.9
JF26	01-20-2019	-75	969.1	JF47	11-20-2022	-45	1,543.1
JF27	03-20-2019	-83	994.2	JF48	05-20-2023	-20	638.5
JF28	05-20-2019	-95	1,099.0				

This table summarizes the features of JFs considered in our empirical analysis. "α" refers to a quoted margin expressed in basis points. "Amt issued" is the total amount issued expressed in billion yen.

Table 2: **Pricing Model Errors (JFs)**

	$\beta = 0.1,\ \upsilon = .025$			$\beta = 0.3,\ \upsilon = .025$			$\beta = 0.1,\ \upsilon = .050$			$\beta = 0.3.\ \upsilon = .050$		
τ	Sim	Model	Error	Sim	Model	Error	Sim	Model	Error	Sim	Model	Error
1	99.44	99.44	0.00	99.90	99.90	0.00	100.24	100.24	0.00	101.17	101.17	0.00
2	98.89	98.89	0.00	99.45	99.45	0.00	100.34	100.35	-0.01	101.77	101.77	0.00
3	98.35	98.34	0.00	98.81	98.81	0.00	100.34	100.36	-0.02	101.95	101.96	0.00
4	97.79	97.79	0.00	98.06	98.06	0.00	100.27	100.28	-0.01	101.85	101.86	-0.01
5	97.24	97.23	0.00	97.26	97.26	0.00	100.12	100.13	0.00	101.55	101.56	0.00
6	96.66	96.67	-0.01	96.44	96.44	0.00	99.91	99.91	0.01	101.13	101.13	0.00
7	96.10	96.10	0.00	95.61	95.61	0.00	99.64	99.63	0.01	100.62	100.62	0.00
8	95.52	95.53	-0.01	94.78	94.78	0.00	99.33	99.32	0.01	100.06	100.06	0.00
9	94.94	94.95	-0.02	93.97	93.97	0.00	98.98	98.97	0.01	99.48	99.48	0.00
10	94.35	94.38	-0.03	93.17	93.17	0.00	98.59	98.59	0.00	98.89	98.89	0.00
11	93.78	93.81	-0.03	92.39	92.39	0.00	98.19	98.19	0.00	98.31	98.30	0.00
12	93.22	93.24	-0.02	91.62	91.62	0.00	97.78	97.78	0.00	97.73	97.72	0.01
13	92.64	92.66	-0.02	90.87	90.88	0.00	97.36	97.36	0.00	97.17	97.16	0.01
14	92.07	92.10	-0.03	90.15	90.15	0.00	96.93	96.93	0.00	96.62	96.61	0.01
15	91.51	91.54	-0.03	89.43	89.44	0.00	96.50	96.50	0.01	96.10	96.09	0.01

This table reports the fair values of JFs under the different values of β, υ, and τ (time-to-maturity). "Sim" is the fair value computed by simulation whereas "Model" refers to the value computed by the approximate analytical solution. ϵ, a volatility term, is fixed at 0.009. Without loss of generality, we assume that the market price of risk is zero.

Table 3: **Descriptive Statistics of Liquidity Discounts in the JF Prices**

	N	Avg.	Std.	Max (date)	Min (date)		N	Avg.	Std.	Max (date)	Min (date)
JF8	100	-0.68	1.93	2.37 [2004-12]	-6.12 [2008-03]	JF29	74	-3.05	3.14	2.04 [2004-12]	-9.09 [2008-10]
JF9	100	-0.94	2.22	2.33 [2004-12]	-6.37 [2008-03]	JF30	72	-3.28	3.15	2.02 [2004-12]	-9.35 [2008-10]
JF10	100	-1.06	2.30	2.35 [2002-09]	-6.72 [2008-03]	JF31	70	-3.55	3.08	1.73 [2005-03]	-9.57 [2008-10]
JF11	100	-1.21	2.48	2.42 [2002-09]	-6.85 [2008-03]	JF32	68	-3.65	2.96	1.77 [2005-03]	-9.52 [2008-10]
JF12	100	-1.23	2.46	2.50 [2002-09]	-7.04 [2008-03]	JF33	66	-3.84	2.87	1.55 [2005-05]	-9.61 [2008-12]
JF13	100	-1.44	2.63	2.79 [2002-07]	-7.56 [2008-11]	JF34	63	-4.15	2.76	0.83 [2005-10]	-9.65 [2008-12]
JF14	100	-1.45	2.64	2.80 [2002-07]	-7.64 [2008-11]	JF35	62	-4.20	2.75	0.99 [2005-10]	-9.83 [2008-10]
JF15	100	-1.52	2.74	3.02 [2002-07]	-7.70 [2008-11]	JF36	60	-4.44	2.69	-0.37 [2005-11]	-10.49 [2008-12]
JF16	100	-1.62	2.80	3.21 [2002-07]	-7.84 [2008-11]	JF37	57	-4.63	2.62	-1.03 [2006-04]	-9.93 [2008-10]
JF17	98	-1.70	2.72	2.58 [2002-09]	-7.42 [2008-11]	JF38	56	-4.43	2.47	-0.88 [2006-04]	-9.18 [2008-10]
JF18	96	-1.80	2.78	2.06 [2004-12]	-7.48 [2008-11]	JF39	53	-4.25	2.22	-0.79 [2007-04]	-8.21 [2008-03]
JF19	93	-1.97	2.90	2.12 [2005-03]	-7.68 [2008-11]	JF40	52	-4.22	2.21	-0.71 [2006-12]	-7.82 [2008-03]
JF20	92	-2.01	2.96	2.19 [2003-04]	-7.71 [2008-11]	JF41	50	-4.24	2.18	-0.63 [2006-12]	-8.11 [2008-10]
JF21	89	-2.08	2.92	2.36 [2005-03]	-7.75 [2008-11]	JF42	48	-4.41	2.12	-0.73 [2006-12]	-8.03 [2008-10]
JF22	88	-2.17	3.02	2.49 [2005-03]	-7.59 [2008-10]	JF43	45	-4.66	1.98	-0.77 [2007-04]	-7.84 [2008-10]
JF23	86	-2.18	3.00	2.43 [2005-03]	-7.79[2008-10]	JF44	41	-5.17	1.75	-0.85 [2007-06]	-7.84 [2008-10]
JF24	84	-2.27	3.07	2.49 [2005-03]	-7.91 [2008-10]	JF45	38	-5.52	1.37	-2.59 [2007-09]	-7.79 [2008-10]
JF25	82	-2.41	3.12	2.23 [2005-03]	-8.09 [2008-10]	JF46	36	-5.31	1.23	-2.61 [2007-11]	-7.66 [2008-03]
JF26	80	-2.55	3.19	2.30 [2005-03]	-8.51 [2008-10]	JF47	29	-5.95	0.98	-3.02 [2010-08]	-7.62 [2008-10]
JF27	78	-2.72	3.19	2.08 [2004-12]	-8.79 [2008-10]	JF48	26	-5.50	0.97	-2.62 [2010-08]	-6.75 [2009-12]
JF28	76	-2.93	3.17	1.96 [2004-09]	-9.02 [2008-10]						

This table tabulates the descriptive statistics of liquidity discounts defined as "the market prices of JFs minus their corresponding fair values." "N" is the number of observations, and 'Avg.' and 'Std.' are the average and standard deviation of liquidity discounts, respectively.

Table 4: **Bayesian Model Comparison**

	Log likelihood	Marginal Likelihood	Posterior Prob.of Model
Non-switching	5342.56	5237.92	0.00
Regime-switching	7737.00	7734.19	1.00

This table reports the log likelihood and the log marginal likelihood values of the Non-switching model and the Regime-switching model along with the corresponding posterior probability of each model.

Table 5: **Posterior Distribution of the Regime-dependent Parameters**

	Regime 1 ($s_t = 1$)				Regime 2 ($s_t = 2$)			
	mean	2.50%	97.50%	ineff.	mean	2.50%	97.50%	ineff.
κ_{s_t}	0.40	0.36	0.44	7.59	0.39	0.34	0.43	4.70
μ_{s_t}	0.00	-	-	-	0.07	0.05	0.08	82.00
ω_{s_t}	0.01	0.01	0.02	14.75	0.03	0.02	0.04	11.75
λ_{s_t}	0.01	0.00	0.06	87.19	0.00	-0.05	0.01	69.24
$10^3\sigma^2_{s_t}$	0.89	0.84	0.96	76.01	0.89	0.84	0.96	62.57
π_{s_t,s_t}	0.99	0.97	1.00	87.97	1.00	0.97	1.00	77.60

This table reports the structural parameters of the regime-switching model. "ineff." refers to the inefficiency factor of the parameter, which is computed as equation (3.10).

Table 6: **Posterior mean and 95% credibility intervals of the parameters and changepoints**

Panel A: LDR

	state 1 ($q_t = 1$)		state 2 ($q_t = 2$)	
	mean	95% interval	mean	95% interval
c_{q_t}	0.00	[-0.11, 0.11]	1.15	[0.89, 1.49]
g_{q_t}	0.72	[0.53, 0.90]	0.78	[0.68, 0.89]
v_{q_t}	0.01	[0.01, 0.01]	0.03	[0.02, 0.03]

Panel B: TED spread

	state 1 ($q_t = 1$)		state 2 ($q_t = 2$)	
	mean	95% interval	mean	95% interval
c_{q_t}	0.20	[0.10, 0.32]	0.59	[0.34, 0.85]
g_{q_t}	0.67	[0.39, 0.93]	0.74	[0.62, 0.86]
v_{q_t}	0.01	[0.01, 0.01]	0.04	[0.03, 0.05]

Panel C: JPTED spread

	state 1 ($q_t = 1$)		state 2 ($q_t = 2$)	
	mean	95% interval	mean	95% interval
c_{q_t}	0.18	[0.09, 0.28]	0.93	[0.43, 1.49]
g_{q_t}	0.56	[0.18, 0.93]	0.89	[0.80, 0.97]
v_{q_t}	0.01	[0.01, 0.01]	0.02	[0.02, 0.02]

Panel D: Changepoint

	mean	95% interval
LDR	October 2007	[August 2007, October 2007]
TED	June 2007	[February 2007, August 2007]
JPTED	March 2007	[November 2006, August 2007]

This table summarizes the posterior distributions of the state-dependent parameters and changepoints in equation (4.1). In order to avoid the numerical problem, we use liquidity measures multiplied by 20. These are based on 5,000 simulated posterior draws.

Table 7: **Granger-Causality Analysis**

Panel A: LDR

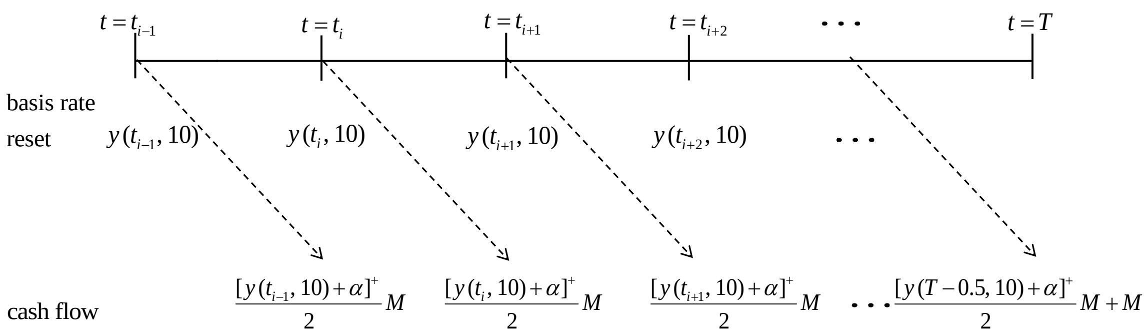
	July 2002 - February 2007			October 2007 - October 2010		
	mean	95% intervals	Prob.(>0)	mean	95% intervals	Prob.(>0)
constant	-0.34	[-0.89, 0.23]	0.16	1.46	[0.37, 2.49]	0.99
ϕ_1	0.76	[0.56, 0.95]	1.00	0.77	[0.60, 0.95]	1.00
ϕ_2	0.10	[-0.27, 0.47]	0.68	0.20	[-0.06, 0.45]	0.91
ϕ_3	0.28	[-0.24, 0.79]	0.81	-0.16	[-0.42, 0.10]	0.15

Panel B: JPTED spread

	July 2002 - February 2007			October 2007 - October 2010		
	mean	95% intervals	Prob.(>0)	mean	95% intervals	Prob.(>0)
constant	0.26	[0.12, 0.98]	0.99	0.38	[-0.49, 1.25]	0.76
ρ_1	0.09	[-0.09, 0.21]	0.77	0.06	[-0.08, 0.19]	0.74
ρ_2	0.18	[-0.19, 0.39]	0.72	0.30	[0.09, 0.50]	0.99
ρ_3	0.24	[-0.08, 0.69]	0.91	0.66	[0.45, 0.87]	1.00

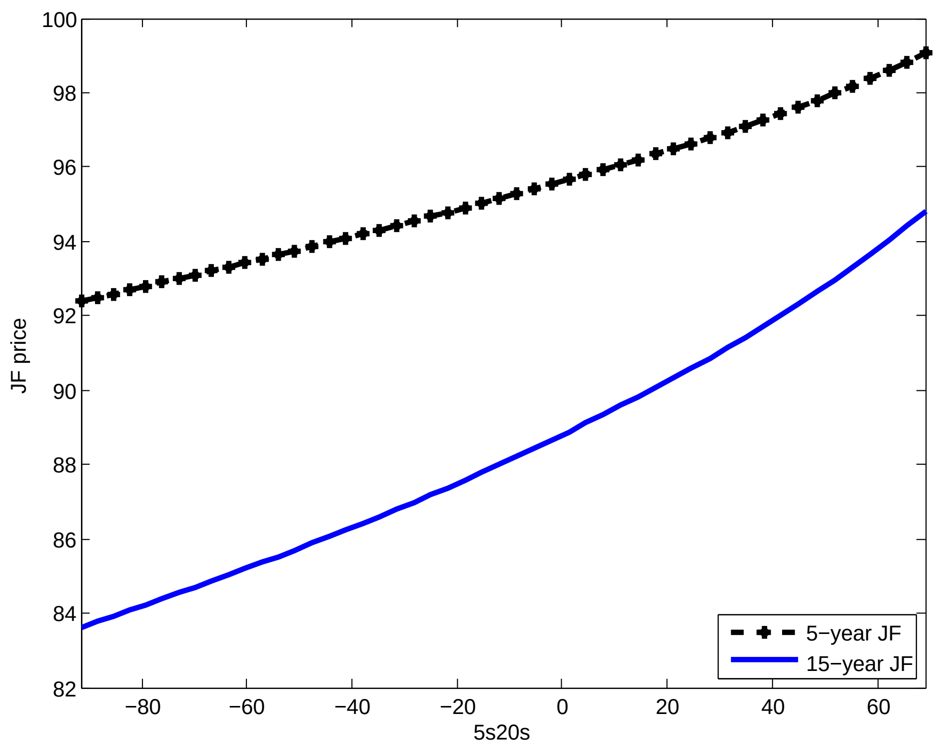
This table presents the Granger-causality results (Equations (4.2) and (4.3)). Prob.(> 0) indicates the posterior probability that each parameter is strictly positive. These are based on 5,000 simulated posterior draws.

Figure 1: **Cash flow Stream of a JF**



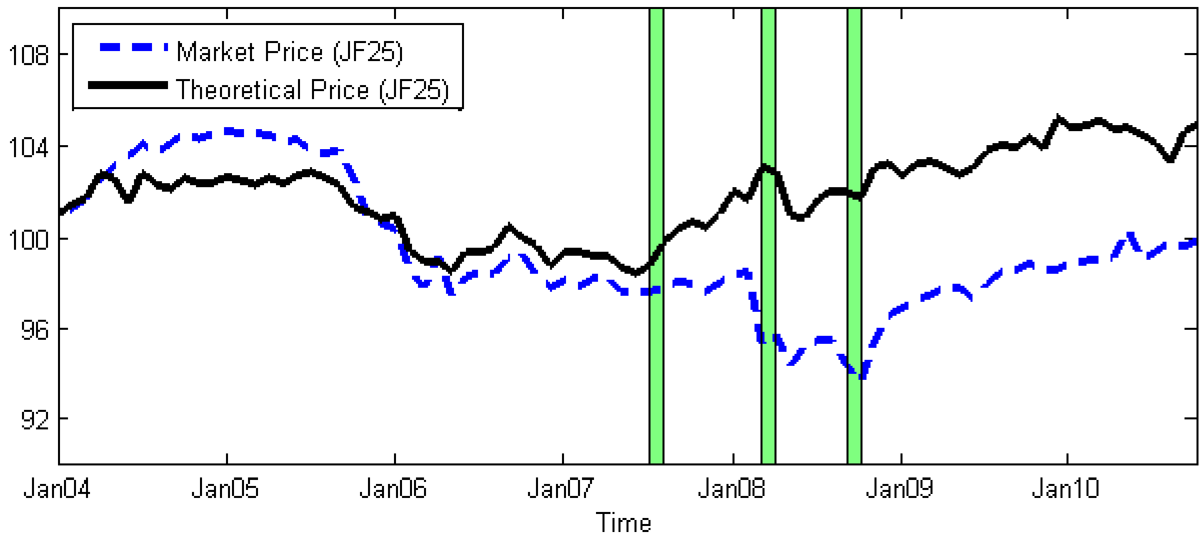
This figure illustrates the cash flow stream of a generic JF at $t = t_i$, where t_j is each coupon payment date and $y(t, 10)$ is the 10-year par yield observed at time t. α and M refer to the quoted margin and the maturity value, respectively.

Figure 2: **Yield Slope vs. JF Prices**



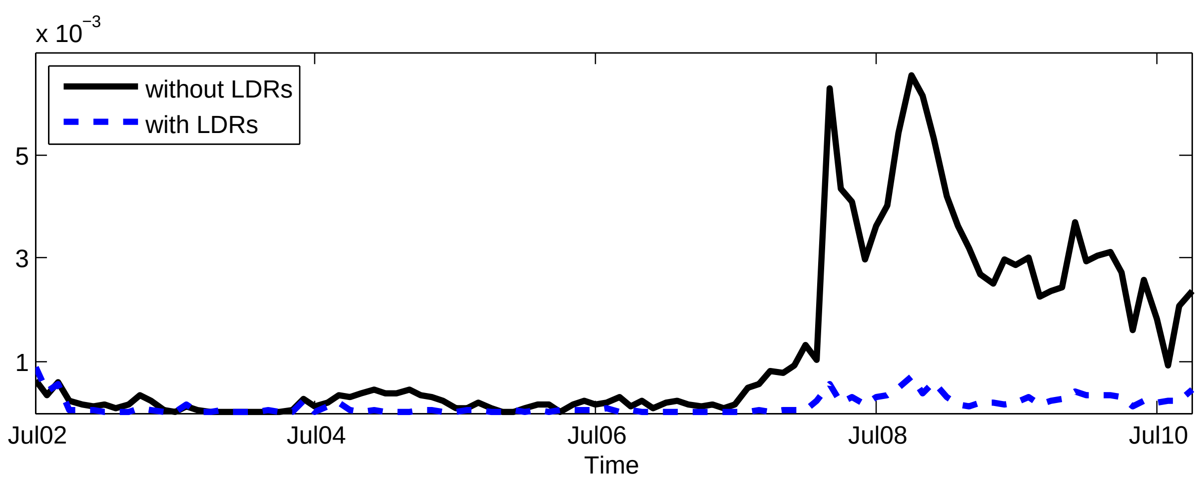
This figure shows the relationship between the spreads of 5-year and 20-year par yields (5s20s) and JF prices.

Figure 3: **Market Price vs. Theoretical Price**



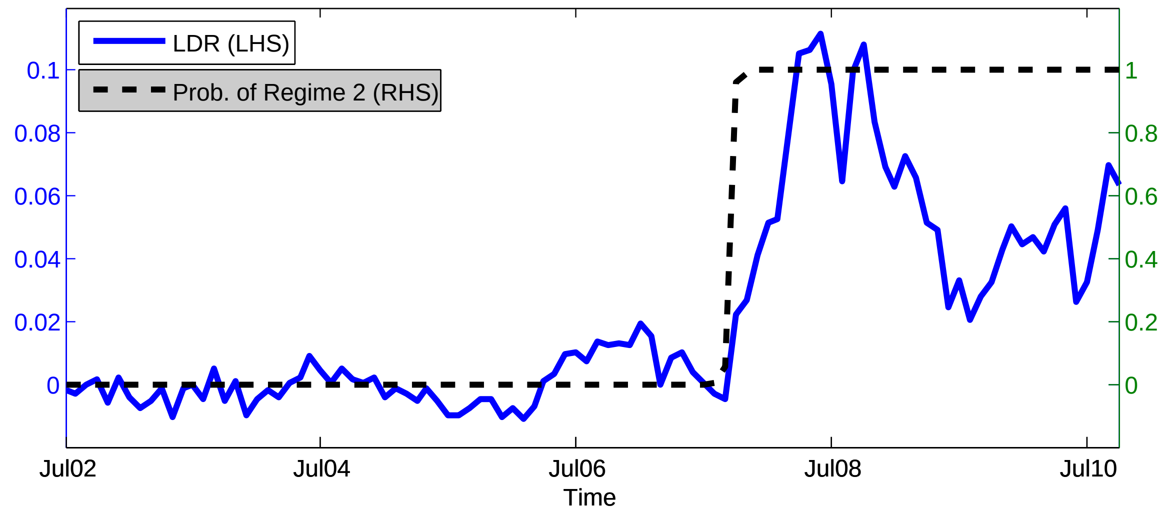
This figure shows the time-series evolution of the market price and theoretical price of JF25 from January 2004 to October 2010. The three shaded areas match critical event periods: BNP Paribas' suspension of withdrawals from three investment funds, acquisition of Bear Stearns by J.P. Morgan, and collapse of Lehman Brothers.

Figure 4: **Pricing Performance with LDR**



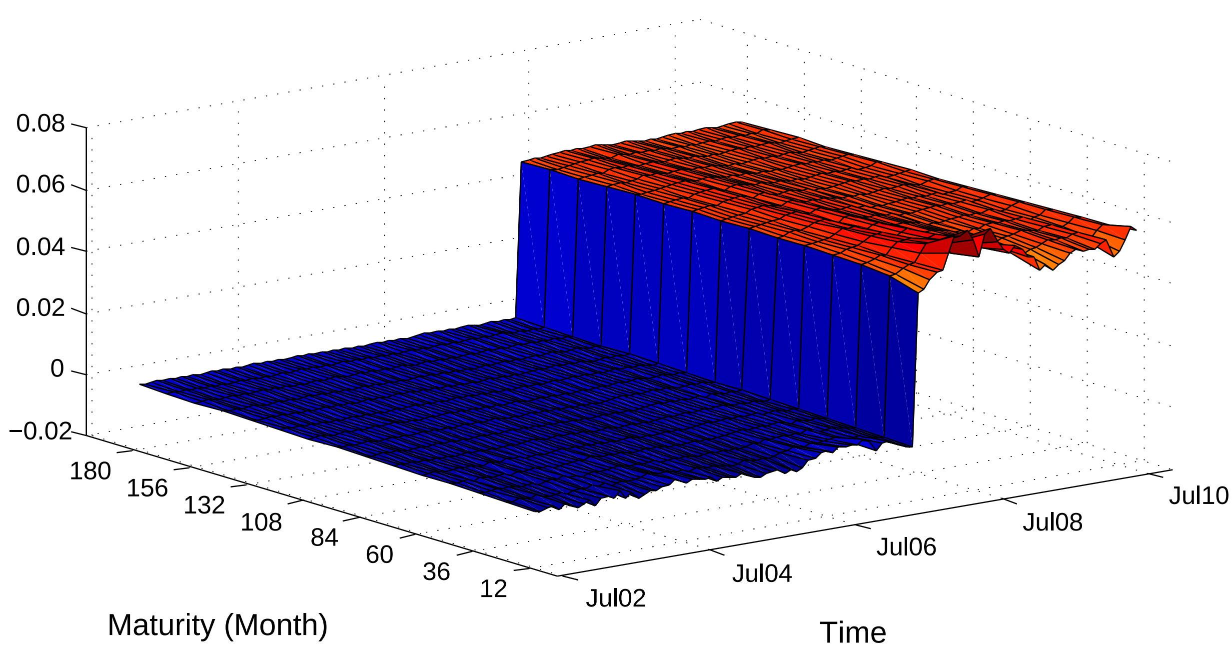
This figure depicts the time-series evolution of price differentials of JFs. The price differentials without LDRs are measured by averaging the sum of squared log differences between market prices and fair values at each point in time. In contrast, those with LDRs are measured by averaging the sum of squared log differences between market prices and corresponding theoretical values of the regime-shifting model.

Figure 5: **Time-series of the Estimated LDR and the Posterior Probability of Regime 2**



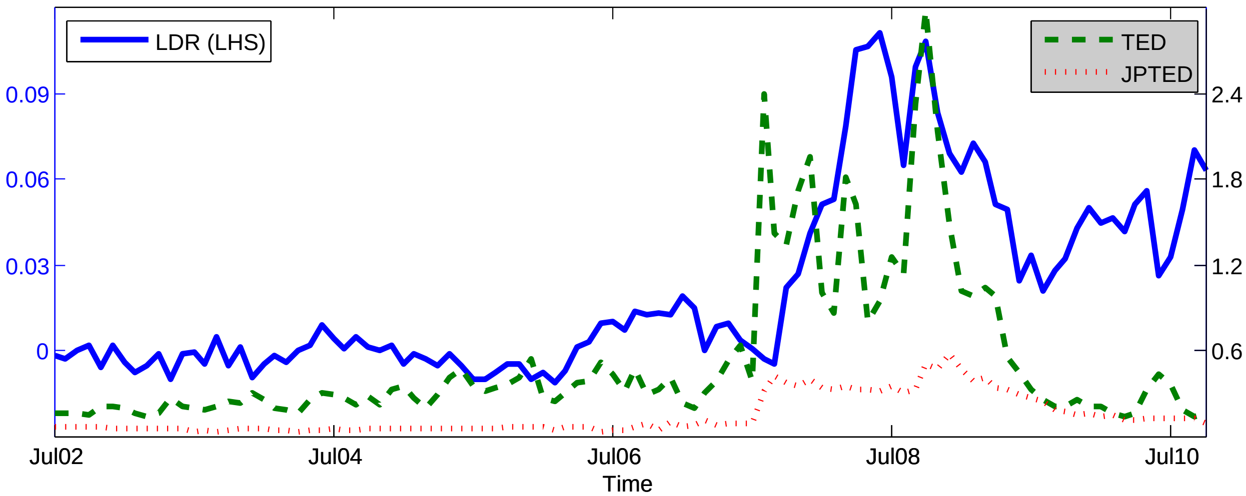
This figure presents the time-series evolution of the estimated LDR and the posterior probability of Regime 2 (the crisis regime).

Figure 6: **Retrieval of the Term Structure of the LDYs**



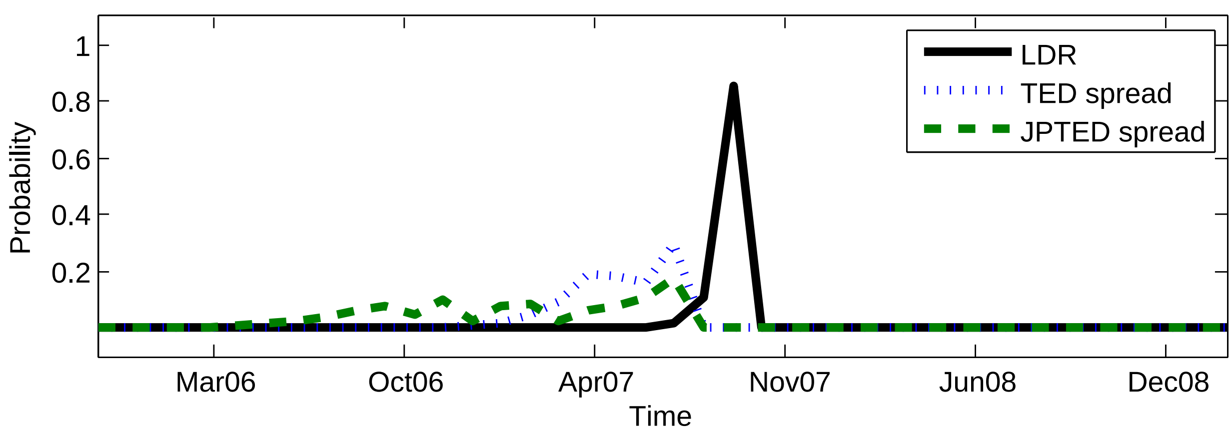
This figure shows the retrieved term structure of LDYs under the regime-switching model.

Figure 7: **Market Illiquidity vs. Funding Illiquidity**



This figure illustrates the time-series dynamics of market illiquidity measured by LDR, global dollar funding illiquidity measured by the U.S. TED spread (TED), and the local yen funding illiquidity measured by the Japanese TED spread (JPTED).

Figure 8: **Posterior Probabilities of the Changepoints**



This figure presents the posterior probabilities of the changepoints of the three illiquidity measure processes: market illiquidity (LDR), dollar funding illiquidity (TED spread), and yen funding illiquidity (JPTED spread). These probabilities are obtained by averaging the changepoints simulated from the MCMC cycles.