Higher education decision and private tutoring for college admission:

theory and structural estimation using Korean panel data*

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Abstract

This paper analyzes dynamic decisions of parents who send their children to college. It first augments the Caucutt and Kumar (2003) dynamic model with private tutoring decision which is more suitable for the South Korean case. According to the model, parents with higher wage would send their children to college even if their *ex ante* admission probability is low as they can raise the probability through private tutoring.

Then it estimates structural parameter such as elasticity of intergenerational substitution and intergenerational discount factor through simulation-based indirect inference designed by Gourieroux, Monfort, and Renault (2003) using Korean education

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and employment panel data. The result implies severe inequality of education in terms of private tutoring due to high elasticity of intergenerational substitution despite a great number of applications to college because of high intergenerational discount factor. This shows that education inequality in the country is revealed more primarily in the level of expenditure on private tutoring rather than the decision to send their children to college.

Based on the estimates, this paper analyzes effects of need-based subsidy scheme like State Scholarship in Korea based on model-based indicators. The results report that the subsidy scheme would increase mean value of individual welfare and improve equality of opportunity, even though it may harm education efficiency.

Keywords: education inequality, private tutoring, college admission, higher education subsidy, structural estimation, Korean education

1 Introduction

College admission and higher education policies are often at the heart of political concerns in South Korea. At the same time, excessive enthusiasm over higher education often ranks Korea as the country that spends the largest amount of GDP on private education and records the highest tertiary education entrance rate among OECD members. This phenomenon bases its roots on the country's desire for equality of opportunity. It has been appreciated to contribute to the country's high growth but now it provokes concerns that it may harm education efficiency and national productivity in the long run.

Compared to its importance, higher education decisions of parents on their children have rarely been dealt with through an economic model. The analysis of higher education was often anecdotic, qualitative or purely statistical especially in the literature of South Korea. Thus this paper elaborates higher education issues in South Korea using dynamic decision model of parents. It understands college education decision of parents as an equilibrium outcome of dynamic programming problem given heterogeneity of parents' wages and children's abilities. At the same time, by estimating structural parameters of the model, it explains major characteristics of South Korean higher education and elaborate welfare implications of higher education subsidy scheme.

This paper analyzes dynamic education decision of parents by modifying Caucutt and Kumar (2003) overlapping generation model to fit the Korean actualities. It adds the possibility of parents to pay for private tutoring to make their children more likely to receive an admission from college. Results of the model imply students with richer parents are more likely to get higher level of private tutoring, and get a college admission even when they have lower abilities. Furthermore, substitution effect between private tutoring and ability makes students with higher ability tend to get smaller amount of private tutoring.

Then it estimates structural parameter of this model using simulation-based indirect inference with Korean panel data on high school students and their parents. It tries to find model parameter that minimizes the distance between auxiliary model estimators from samples and those from simulated data. Then parameter estimates are used to derive implications on schemes of need-based higher education subsidy in Korea.

2 Literature Review

Since 2000s, literature in economics on education inequality has focused on existence, cause and effects of intragenerational credit constraint, which bases itself on poor information or financial frictions.

Hanushek and Leung (2014) constructs a general equilibrium model which deals with dynamic inefficiency under credit constraint on human capital accumulation when parents can bequest to the children. Then it explains general equilibrium effects of governmental subsidy on tertiary education. Subsidy schemes have different effects on the welfare of economy when it is need-based and merit-based.

Lochner and Monge-Naranjo (2011) derives endogenous credit constraint on human capital accumulation with student loan structure and limited repayment incentive in private loan market. It further reports that increase in model-implied credit constraint is empirically associated with the growing effect of parents' economic power on college entrance of their children.

On the top of that, Caucutt and Kumar (2003) is one of the first paper which treats higher education decision as a result of dynamic concern to maximize inter-generational utility. It derives a numerical solution to the dynamic optimization problem with dynamic programming. According to the paper, when there exists heterogeneity in the wage of parents and the ability of children under intra-generational credit constraint without bequest, parents' wage affects how important the ability of children is in deciding whether to send their children to college. In other words, when parents are richer, they send their children even if their children have lower abilities but when parents are poorer, they send their children only when their children have higher abilities. In this structure, Caucutt and Kumar (2003) analyzes how government subsidy scheme on higher education affect college entrance, welfare and expected education cost. Because of its applicable features, this paper aims to develop and estimate the model of Caucutt and Kumar (2003).

Researches on education inequality in Korea are usually based on empirical analysis using statistical models. Chang (2000) shows that effects of parents' economic power on college entrance of their children have been decreasing, using OLS and LOGIT regression of parents' wealth onto the final academic years achieved by their children and the college entrance respectively. Based on the regression results, he points out that the effects parents' wealth has on the education of their children are not structural, thus they turn out to be unstable across time. Like this, unstable relationship between variables deepen the necessity to design a structural model about parents' economic incentive for education of their children.

Yeo, Kim, Ku, and Kim (2007) deals with statistical facts on education inequality of Korean society after independence. It states that higher education was expanded due to standardization policies of middle school and high school in 1968 and in 1973 as well as liberalization of establishing college and setting maximum number of students in college in 1995. However, dissatisfaction with public secondary education led to expansion of private tutoring for college admission, which contributed to the increase in education inequality. According to the paper, poverty rate of the group with low academic attainment increased despite rise of academic achievement, and this seems to be derived from increase in income inequality rather than changes in income distribution.

This paper suggests several facts which can be employed to model dynamic decision of parents who decide academic attainment of their children in Korean context. Since 1990s, inequality of individual income has been caused more significantly by income difference within groups than between groups but as the group gets younger, income difference between groups becomes bigger. What seems remarkable is the relationship between income of parents and the score of the Scholastic Aptitude Test of their children is statistically insignificant or very weak. Furthermore, entrance rate of tertiary education is not substantially different across income classes. However, when it comes to four-year full time college, there exists a gap in entrance rate across economic capacities of parents. On the top of that, socio-economic status of parents and that of their children have apparent correlation and it seems this relationship is diffused with education attainment.

The model that would be constructed below bases its structure on the facts above. It would assume that the ability of children and that of parents are distributed independently, and richer parents can increase admission probability of their children with private tutoring. Moreover, there exists substitutability between children's ability and private tutoring and the model assumes admission probability is concave with respect to these two factors. Thus it observes similar probability of admission to college between students from poor family with higher abilities and students from rich family with lower abilities *ex post* due to private tutoring of the latter. Finally, we limit the way that parents can affect the future utility of their children into higher education and higher education is interpreted as four-year full time college.

3 Model

The model basically follows the Caucutt and Kumar (2003) overlapping generation model. But the model is modified by adding heterogeneous wage even in the same education level of parents and parents' option to choose an expenditure level on private tutoring.

Agents live two periods. In the first period, they are educated in school and sent further to college upon the parents' decision. If they are sent to college, they can receive an admission from the college with probability $\pi(a, \rho)$, where a denotes ability of their children and ρ expenditure level on private tutoring. If they are admitted, they would graduate the college with probability 1. This is different from Caucutt and Kumar (2003) which assumes the probability of graduation, not admission that varies from high-able to low-able students. This is because the Korean universities has extremely lower drop-out or flunk rate but the admission is relatively severe.

In the second period, they become parents, bear a child, and decide whether to send their children to college and how much to spend on private tutoring of their children given their heterogeneous wage and ability of their children.

The exact sequencing of parents' decision in the second period is

1. observe realized wage and ability of children.
2. decide whether to send his child to the college by comparing expected values of each case.
3. choose how much to spend on private tutoring.
4. determine how much to consume from what is left from former decisions.

Then the parents' Bellman equation is given as,

$$V(w,a) = \max(V_c(w,a), V_s(w,a)) \tag{1}$$

V_c is the expected utility that parents perceive when they decide to send their children to college, but V_s is one that they perceive when they do not decide to do so. Then V_c and V_s are expressed as the following equations.

$$V_c(w,a) = \max_{\rho}\{u(w-e-\rho) + \beta\pi(a,\rho)\mathbf{E}_{c,a'}V(w'_c,a') + \beta(1-\pi(a,\rho))\mathbf{E}_{s,a'}V(w'_s,a')\} \tag{2}$$

and

$$V_s(w,a) = u(w) + \beta\mathbf{E}_{s,a'}V(w'_s,a') \tag{3}$$

where $\mathbf{E}_{c,a'}V(w'_c,a')$ is the expected utility when the child gets a college admission, $\mathbf{E}_{s,a'}V(w'_s,a')$ when the child remains a school-educated worker. w, u, e, a, ρ, π, β denote wage, utility, fixed cost of college education decision, ability of children, expenditure on private tutoring, college admission probability and intergenerational discount factor, respectively.

Note that the parents know completely the ability of their children when they determine the education and parents' ability does not affect ability distribution of their children. Thus expected value of future utility does not change across the ability of their children as it only depends on the stochastic ability of their grandchildren. Ability of children contributes only to higher admission probability when they are sent to college.

The utility function $u(c)$ and college admission probability function $\pi(a,\rho)$ are assumed to follow standard conditions of optimization problem.

Assumption 1 *$u'(c) > 0$ and $u''(c) < 0$.*

Assumption 2 *$\partial\pi(a,\rho)/\partial a > 0$, $\partial\pi(a,\rho)/\partial\rho > 0$ and $\partial^2\pi(a,\rho)/\partial\rho^2 < 0$.*

There is another property the college admission probability function satisfies; ability and private tutoring substitutes each other for college admission.

Assumption 3 *$\partial^2\pi(a,\rho)/\partial a\partial\rho < 0$.*

The benefit of being college educated comes from higher probability of earning high wage. More specifically, the wage distribution of college educated first-order stochastically dominates that of school educated and the ability and wage are jointly independent, as Assumption 4 says.

Assumption 4 *$F_{c,a}(w,a) = F_c(w)F_a(a)$, $F_{s,a}(w,a) = F_s(w)F_a(a)$ and $F_c(w) < F_s(w) \quad \forall w \in \mathcal{R}^+$, where $F_{c,a}, F_{s,a}, F_c, F_s, F_a$ denote the joint cumulative distribution function of wage of college educated and ability, wage of school educated and ability, the marginal cumulative distribution function of wage of college educated, school educated and ability, respectively.*

Parents internalize benefits of their children due to higher education with intergenerational discount factor β. For simplicity of the model, there is no way to affect the expected utility of their children other than higher education, like Caucutt and Kumar (2003) and other overlapping models in the literature. Thus parents who want to increase the expected value of their children can achieve their goal only with higher education. To achieve this, they have to pay a fixed cost, e and some costs, upon their decision, of private tutoring, ρ.

Unlike Caucutt and Kumar (2003), the wage structure of this economy does not depend on the ratio of college educated to the school-educated. This would simplify the discussion but does not harm the reliability of this model to analyze South Korean society. As the country is small open economy and dependent on export to a great degree, the factor price of this economy does not depend on the domestic ratio of college-educated people under the presence of highly free trade, as suggested by the famous Stolper-Samuelson theorem.

This feature simplifies the model, as the original Caucutt and Kumar (2003) dynamic model necessitates rational expectation assumption since their behavior on higher education as a whole affects the environment in the future. This model assumes the wage structure in the current situation would hold in the future.

Then some of the basic results of Caucutt and Kumar (2003) apply as well, with details slightly modified.

Proposition 1 *In any equilibrium in which parents behave optimally,* $\mathbf{E}_{c,a'}V(w'_c, a') > \mathbf{E}_{s,a'}V(w'_s, a')$*, where* $\mathbf{E}_{c,a'}V(w'_c, a')$ *and* $\mathbf{E}_{s,a'}V(w'_s, a')$ *mean expected value integrated by* $F_{c,a'}$ *and* $F_{s,a'}$*, respectively.*

Proposition 2 *For any given* w*, there exist unique* $a^*(w)$ *such that a parent with* w *sends his child to college if* $a \geq a^*(w)$ *and does not otherwise.*

Furthermore, we can conclude that once decided to apply for a college, children with higher ability would receive lower level of private tutoring, given wage is fixed and children in a richer family would get higher level of private tutoring, given ability is fixed.

Proposition 3 *Suppose* $a_1 < a_2$*. Then, for a fixed* w*,* $\rho_1 \geq \rho_2$*, where* ρ_1 *and* ρ_2 *are the maximizing level of private tutoring of* a_1 *and* a_2*, respectively.*

Proposition 4 *Suppose* $w_1 < w_2$*. Then for a fixed* a*,* $\rho_1 \leq \rho_2$*, where* ρ_1 *and* ρ_2 *are the maximizing level of private tutoring given* w_1 *and* w_2*, respectively.*

The proofs of propositions can be found in the appendix.

However, unlike Caucutt and Kumar (2003), we cannot be sure that students in richer family would have smaller threshold ability level, due to the effect of wages and abilities on private tutoring. From the definition of a^*,

$$u(w) - u(w - e - \rho) = \beta\pi(a^*(w), \rho)(\mathbf{E}_{c,a'}V(w'_c, a') - \mathbf{E}_{s,a'}V(w'_s, a')) \tag{4}$$

From total differentiation and implicit function theorem,

$$\begin{aligned} \tfrac{da^*}{dw} \quad = \quad & [u'(w-e-p)\tfrac{\partial\rho}{\partial a} + \beta(\mathbf{E}_{c,a'}V(w'_c, a') - \mathbf{E}_{s,a'}V(w'_s, a'))(\pi_a(a^*, \rho) \\ & +\pi_\rho(a^*, \rho)\tfrac{\partial\rho}{\partial a})] \\ \div \quad & [u'(w) - u'(w-e-\rho) + u'(w-e-\rho)\tfrac{\partial\rho}{\partial w} - \beta\pi_\rho(a^*, \rho)\tfrac{\partial\rho}{\partial w} \\ & \times(\mathbf{E}_{c,a'}V(w'_c, a') - \mathbf{E}_{s,a'}V(w'_s, a'))] \end{aligned} \tag{5}$$

When $\partial\rho/\partial a \leq 0$ and $\partial\rho/\partial w \geq 0$ from proposition 3 and 4, we cannot guarantee that higher wage would lead to lower threshold ability level for children to be sent to college. Note that, when $\partial\rho/\partial a = 0$ and $\partial\rho/\partial w = 0$, $da^*/dw < 0$, as in the standard Caucutt and Kumar (2003) model. However, as shown below, the threshold level tends to be lower when the wage is higher in most of the plausible parameter settings.

3.1 Numerical Examples

Utility function is parameterized as the conventional CRRA function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

The probability of college admission, $\pi(a, \rho)$ is set to be estimated with PROBIT model[1]

$$\pi(a, \rho) = \Phi(\alpha_0 + \alpha_1 a + \alpha_2 \rho)$$

[1] When the argument is higher than 0, the normal cumulative distribution function satisfies assumption 2.

Parameter	Value	Parameter	Value
σ	2	$\mathbf{E}_s(w_s)$	2.4
e	0.02	$\mathbf{E}_c(w_c)$	5
β	0.6	$\mathbf{Var}_s(w_s)$	1
α_0	-3	$\mathbf{Var}_c(w_c)$	2
α_1	1	$\mathbf{E}(a)$	3.5
α_2	2	$\mathbf{Var}(a)$	1

Table 1: Numerical Examples - Parameters

where Φ is the cumulative distribution function of the standard normal distribution.

The value function of this problem is found by value function iteration using linear interpolation with fixed grid points. Then the value function and the policy function are used to find the threshold ability level $a^*(w)$. Calibrated parameters are presented in table 1. Wages and abilities are assumed to be normally distributed with their respective means and variances that are reported in table 1[2].

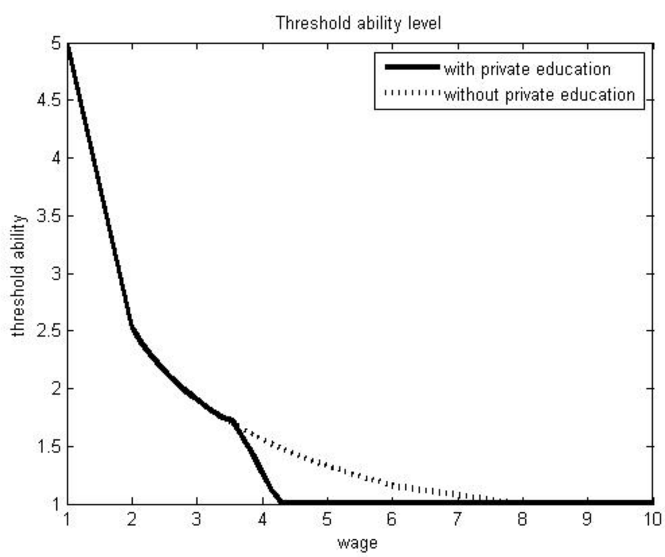

Figure 1: Numerical Examples - threshold ability level

As figure 1 shows, threshold ability level tends to get lower as the wage of parents be-

[2]Wage of 1 in the model and in the data is assume to be equal to monthly income of one million won in South Korea

comes higher. That is, children in richer parents have higher probability of sent to college even if they have lower abilities. As written above, there exists a short interval where the threshold level is strictly increasing, but we can see that it is monotonically decreasing overall. Figure 2 implies children in richer family tend to receive higher level of private tutoring, and children with higher ability have lower level of private tutoring [3].

Existence of private tutoring facilitates richer parents to increase the expected ability of their children. That is to say, students who would not be sent to college without private tutoring are sent and enjoy higher level of college admission due to private tutoring. This deepens the gap of threshold ability level across different economic capacities of parents, such as wage. We can observe this phenomenon by comparing the threshold ability curve with and without private tutoring in figure 1.

Thus the correlation between parents' and children's wage would be positive even when the ability is independently distributed across generations. This harms education efficiency as well as equality opportunity, since children with lower ability but in richer family would have *ex ante* lower college admission probability but end up having higher one owing to private tutoring *ex post*. It would increase expected cost of having a college educated from social planner's perspective.

4 Estimation

Diagnosis of actualities and policy evaluation necessitates estimating proper values of model parameters. Elasticity of intergenerational substitution σ, inter-generational discount factor β and the probability of college completion $\pi(a, \rho)$ are crucial to outcomes but not observ-

[3] When the parents do not decide to send their children to college, their private expenditure level appears to be -1 in the graph.

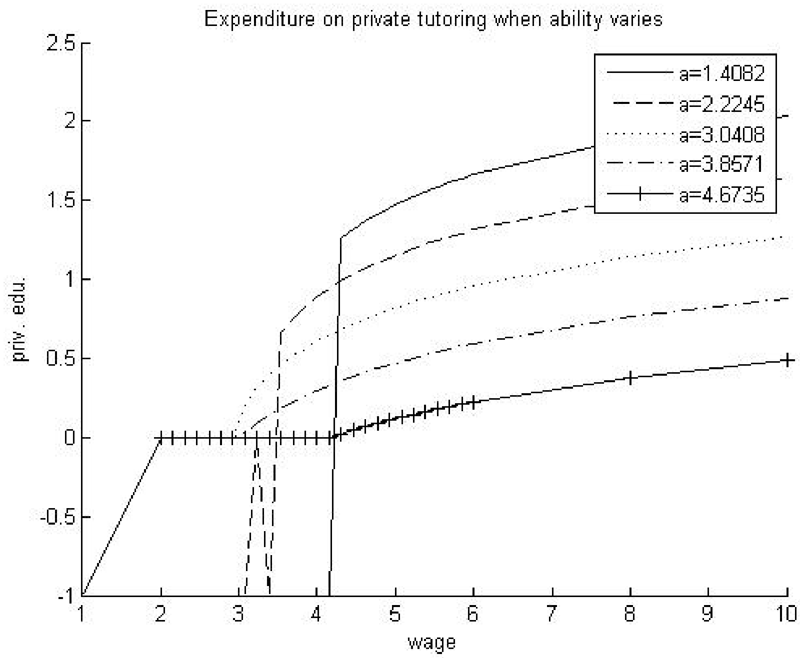

Figure 2: Numerical Examples - expenditure on private tutoring

able. Thus this paper estimates model parameter σ and β using indirect inference based on simulation and induce model implied threshold level of ability across parents with heterogeneous wage. More specifically, it estimates $\pi(a, \rho)$ based on PROBIT estimation in the first stage, then it uses the fitted values to simulate the policy function of parents as a function of their wages. By doing so, it posits that parents making the higher education decision identify the same success probability function as it does in the first stage.

This paper employs Korean Education and Employment Panel Data[4] of made by Korea Research Institute for Vocational Education and Training(KRIVET). The panel data contains 10-year information on secondary and tertiary education of approximately 2000 students and their parents, such as their college education decision, their family income, their grades[5] and their college admission. The data is used to estimate the effect of ability

[4] It uses the cohort of those who were 14 years old in 2004, when the panel data started.

[5] Here it uses the answers to the questions of 'Are you good at literature/ mathematics/ English/ science/ social studies/ arts/ music/ sports?' and 'From first to ninth grade, how would you evaluate your grade last semester?' as a proxy to measure their abilities. The answer to the former is one to five integer scale. It first

and private tutoring on the admission probability in the first stage, and the effect of parents' wage and children's ability on their college education decision in the second stage, which will be described below.

As dynamic programming often does not yield analytic solution, it is difficult to estimate the model parameter with classical methods. Thus simulation based method is employed, especially indirect inference, which is designed by Gourieroux et al.(1993). It tries to match the moments of auxiliary model from data with moments from simulated data. Observed data is used to get an OLS estimate of parents' wage and children's ability on private tutoring, and a LOGIT estimate of them on college education decision. Then for each σ and β in a set of feasible set, it draws simulated panel policy data as a function of observed panel data on state variable added with normally distributed measurement error. With the simulated policy function, it calculates the OLS and LOGIT estimates, which is basically a function of σ and β. The estimator for σ and β would render the distance between the simulated estimates and observed estimates the smallest. The simulation number is fixed at 10 and the objective function is given as

$$\hat{\theta}_{ST} = \arg\min_{\theta} [\hat{\beta}_T - \hat{\beta}_{ST}(\theta)]' \Omega_T [\hat{\beta}_T - \hat{\beta}_{ST}(\theta)] \tag{6}$$

where Ω_T is a weight matrix, $\hat{\beta}_T$ the auxiliary parameter estimator from the observed data and $\hat{\beta}_{ST}$ the mean of auxiliary parameter estimators from simulated data.

We can estimate the optimal weight matrix which minimizes the asymptotic variance of the estimator by following formula.

$$\hat{\Omega}^* = \hat{J}_0 \widehat{(I_0 - K_0)}^{-1} \hat{J}_0$$

calculates the mean value of the largest three numbers of the answers to the first question. And it changes the scale of the answer to the second question into one to five scale. Then it measures the abilities as mean value of the two numbers. Later on, it would add measurement error to this variable.

where $\hat{J}_0 = -\partial^2\psi_T(\theta,\beta)/\partial\beta\partial\beta'$, $\widehat{(I_0 - K_0)}^{-1} = \frac{T}{S}\sum_{s=1}^{S}(W_s - \bar{W})(W_s - \bar{W})', W_s = \frac{\partial\psi_T(\hat{\theta},\hat{\beta})}{\partial\beta}$, $\bar{W} = \frac{1}{S}\sum_{s=1}^{S} W_s$ and $\psi_T(\theta,\beta)$ is the likelihood function of auxiliary model, θ the parameter of original model, β that of auxiliary model, T number of observations and S number of simulations.

In order to get an estimate of optimal weight matrix, the parameters are first estimated with identity weight matrix. Then the estimate is used to calculate the optimal weight matrix. The final estimator of parameters are defined as argument that minimizes the difference in moments, weighted by the optimal matrix.

Inference based on simulation needs long process of calculation. Thus this paper limits the scope of σ and β into the grid within interval [2.00, 4.00] and [0.50, 0.90] with fixed length 0.01 and 0.05, respectively. Then it compares the objective function values calculated on the 1809 grid points then find the minimizer. Other variables are assigned same as table 1. Estimation results and the objective function are presented in table 3 and figure 3.

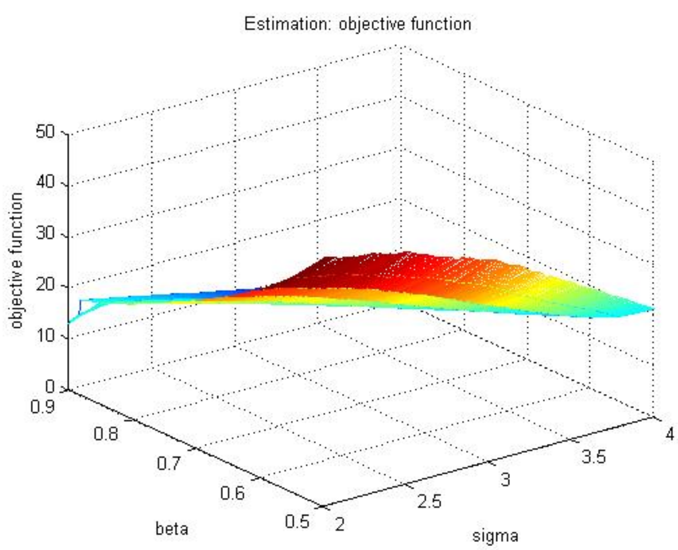

Figure 3: Estimation - Objective function

The most remarkable feature of estimation is the estimator for σ and β minimizes ob-

Parameter	Value
σ	4.00
β	0.90
α_0	-2.7746
α_1	0.8012
α_2	1.5836

Table 2: Estimation - Parameters

jective function at the upper bound 4.00 and 0.90. This shows large gap of private tutoring across different economic capacities of parents in South Korean society as well as the country's high zeal for children's education.

As β is the intergenerational discount factor in this model, high β means parents value more the increase in utility of their children. In other words, they make much of the effect of higher education that increase in the expected income makes their children happier.

σ is the elasticity of intergenerational income substitution and it measures how much the parents are willing to give up in order to make utility of their children higher when their income increases. As σ gets higher, parents tend to invest more on the future utility of their children at the expense of their current utility loss from higher education costs when the income of parents increases. There are two kinds of cost for increasing the expected utility of children. One is the fixed cost of sending children to college, e and the other is variable cost on private tutoring ρ. Thus higher σ incurs more varied reaction of parents to fixed and variable cost of higher education.

These points can be seen in figure 4 and 5. Figure 5 shows how threshold ability level changes and figure 6 shows how the expenditure level of private tutoring changes when σ and β vary. As β gets higher, threshold level from fixed cost becomes lower, thus more students receive higher education. At the same time, private tutoring level is raised as well. When

σ increases, the gap of threshold ability level and that of private tutoring get higher across different wage level of parents.

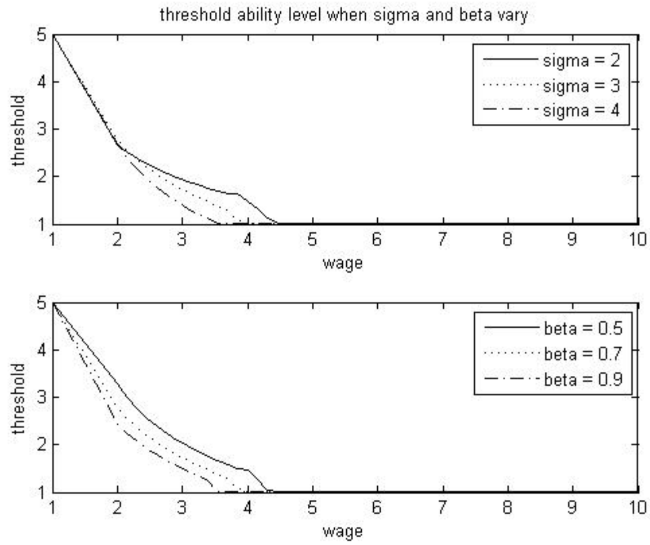

Figure 4: Threshold ability level when sigma and beta vary

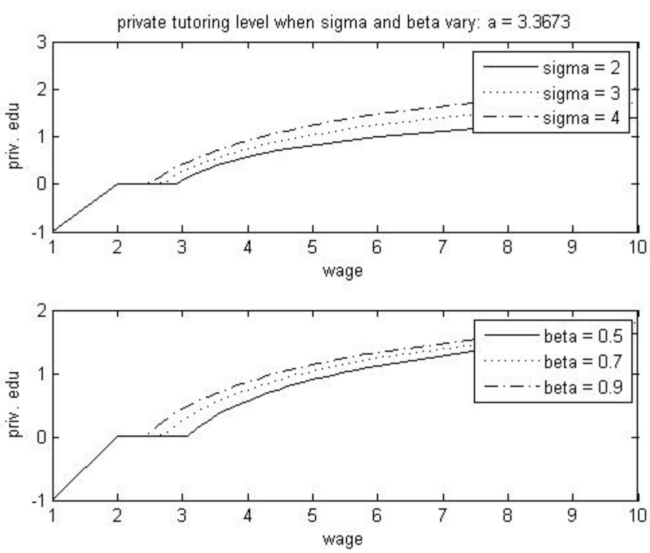

Figure 5: Private tutoring expenditure when sigma and beta vary

Therefore, high estimates of σ and β imply Korea has significantly high level of education inequality due to difference in wealth in spite of high college entrance rate. Most parents want to send their children to college. But there are differences in terms of threshold ability level and expenditure on private tutoring from different economic capacities of parents. Figure 7 and 8 show fitted threshold level and the level of expenditure on private tutoring based on the estimates. As higher σ and β contributes sharply decreasing threshold ability level and most parents earn more than two million won per month, inequality of higher education in Korea is reflected mostly in the private tutoring expenditure, rather than the decision to send children to college.

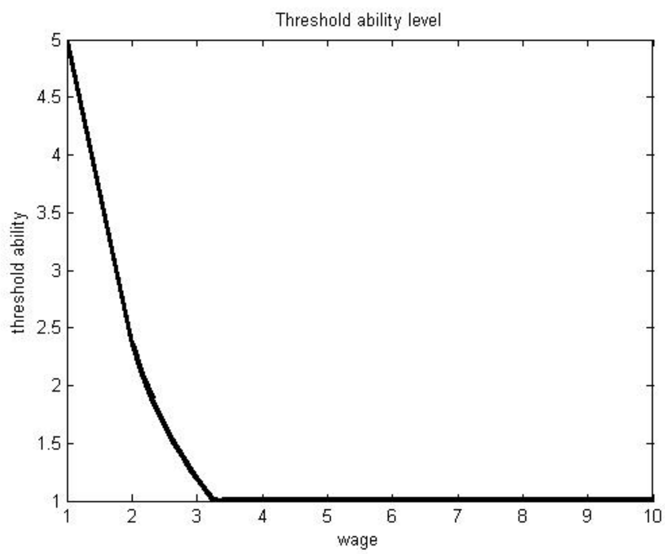

Figure 6: Estimation - threshold ability level

5 Higher Education Subsidy in South Korea

In South Korea, political parties have aroused a controversy over higher education policies in times of election as the equality of opportunity is often interpreted as equal access to higher education. Thus the country has focused on subsidizing higher education through complicated subsidy schemes.

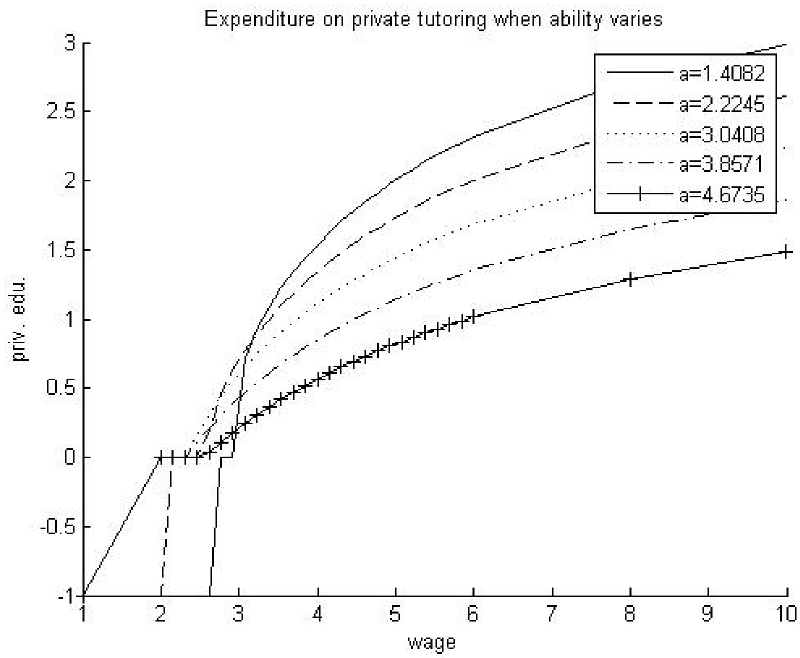

Figure 7: Estimation - expenditure on private tutoring

The vast majority of college education subsidy is implemented by Korea Student Aid Foundation(KOSAF). It has several programs to aid impoverished college students but the main one is the State Scholarship I and II, a need-based aid program paid with grade according to their parents' income. Thus this section aims to analyze effects of need-based scholarship program on some measures based on this model[6].

More specifically, the need-based aid can be interpreted as reducing fixed cost of college education, e of parents who have lower wages into $e - \varepsilon$. The program is financed through labor income tax, of which the rate is denoted by τ. Then the budget constraint of the government implies

[6] As the program has been implemented in a large scale since 2011 and the panel data covers college admissions that started from 2007, it would be fair to assume that the dynamic decision of parents was free from scholarship program when the panel was made.

Measure	Value before subsidy	Value after subsidy
Welfare	-0.4398	-0.0437
Education efficiency	1.0123	1.0020
Equality of opportunity	0.9855	1.0258

Table 3: Policy analysis: measures before and after scholarship

$$\tau \int w_n dG(w_n) = \varepsilon \int a^*((1-\tau)w_n) dG(w_n) \tag{7}$$

where $G(w_n)$ denotes the distribution function of wages in this economy.

Now this paper constructs the following three measures based on the model above to evaluate effects of higher education subsidy.

1. **Welfare**: welfare can be measured as the mean of optimized intergenerational utility of all citizens in this economy.

2. **Education efficiency**: education efficiency can be defined as the inverse of average cost of having one college educated in the society, including fixed cost and variable expenditure on private tutoring.

3. **Equality of opportunity**: this paper measures equality of opportunity by the inverse of standard deviation of threshold ability level across different wage level of parents.

In this paper, we compare above three measures before and after government scholarship program based on the panel data. The scholarship is endowed to families with wages less than 3 and the subsidy level is fixed at 0.05. Then the income tax rate τ is calculated to meet the balanced budget constraint of the government, equation 7, which is equivalent to 0.034%. The result is reported in table 3.

The result emphasizes importance of estimating model parameters properly to derive policy implication. Caucutt and Kumar (2003) reports that need-based scholarship program tends to lower the welfare and education efficiency as the government cannot distinguish students with higher ability and from those with lower ability, and the subsidizing scheme for students in a poor family, regardless of their ability, would provoke loss of efficiency.

However, their concern turned out to be structure- and parameter-specific. By assuming different structure and assigning different values of parameter, even need-based aid program can enhance welfare and equality of opportunity of the economy, although it harms efficiency of higher education. The reason education efficiency decreases is reduction in the fixed cost causes students with the lowest ability to apply for college(refer to figure 8). However, as threshold level of households with monthly income less than 3 is decrease sharply to 1, the measure of education inefficiency improves. Increase in welfare is especially remarkable because high β makes it have higher effects on increasing welfare for households with wage less than 3 to get higher expected utility of sending children to college. But higher σ renders it have smaller effects on decreasing welfare to impose income tax on households with higher wage.

6 Conclusion

This paper analyzes the dynamic problem of parents to send their child to college when there is an option to provide private tutoring to their children for a higher probability of college admission. After modelling such a problem and solving it numerically, it estimates model parameters using simulation based methods. High estimates of elasticity of intergenerational substitution and intergenerational discount factor mean not only generally low threshold level of college application but also large difference of private tutoring expenditure across parents. Then the estimates of parameters are used to quantify the effect of need-based

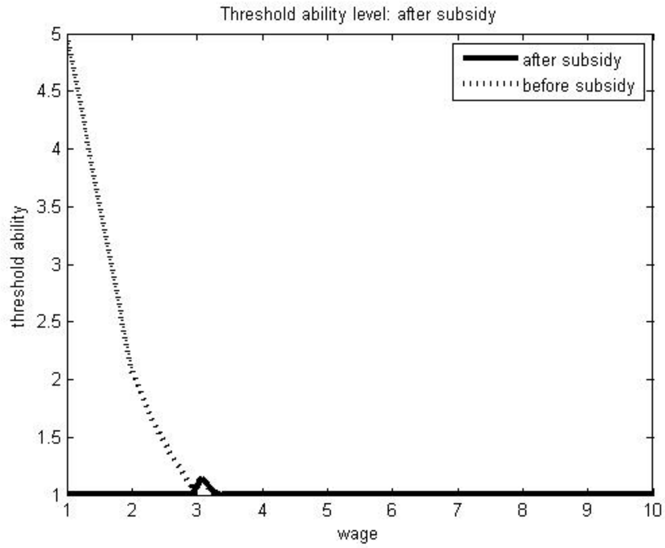

Figure 8: Threshold ability level before and after subsidy

higher education subsidy in South Korean case. Higher education subsidy on low income class would improve average level of welfare as well as equality of opportunity, while it can be harmful to education efficiency in the country.

The framework made in this paper is simple yet useful to be implemented for policy analysis. Some extensions can be made in the analysis of higher education policy based on this model. Though the level of subsidy for college tuition is set to be fixed throughout policy analysis, we can find an optimal level of need-based subsidy level. Then the government is modelled to solve a Ramsey problem, i.e. it tries to maximize sum of welfare in this economy, taking the government's budget constraint and the dynamic decision of parents as given like it is in Caucutt and Kumar (2003). Then we can analyze the effects of optimized need-based aid program with respect to three relevant measures above. We can introduce other subsidy schemes such as merit-based one and compare effects of the policies based on measures defined in this paper. Further research on higher education issues in emerging countries as well as in advanced economies will help us to comprehend social dynamics of mobilities and inequalities in those countries.

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7 Appendix

The proofs of Proposition 1 and 2 were slightly modified from Caucutt and Kumar (2003).

7.1 Proof of Proposition 1

$\mathbf{E}_{c,a'}V(w_c', a') > \mathbf{E}_{s,a'}V(w_s', a')$

Suppose not: then $\mathbf{E}_{c,a'}V(w_c', a') < \mathbf{E}_{s,a'}V(w_s', a')$. We have

$$\beta\pi(a,\rho)\mathbf{E}_{c,a'}V(w_c', a') + \beta(1-\pi(a,\rho))\mathbf{E}_{s,a'}V(w_s', a') < \beta\mathbf{E}_{s,a'}V(w_s', a')$$

and

$$u(w_0 - e) + \mathbf{E}_{c,a'}V(w_c', a') + \beta(1-\pi(a,\rho))\mathbf{E}_{s,a'}V(w_s', a') < u(w_0) + \beta\mathbf{E}_{s,a'}V(w_s', a')$$

for all $w \in \mathcal{R}^+$.

Since all parents do not send their child to college, for all $w_1 < w_2$.

$$V(w_1, a) = u(w_1) + \beta \mathbf{E}_{s,a'} V(w'_s, a') < V(w_2, a) = u(w_2) + \beta \mathbf{E}_{s,a'} V(w'_s, a').$$

However, this equation, approves in turn that $\mathbf{E}_{c,a'} V(w'_c, a') > \mathbf{E}_{s,a'} V(w'_s, a')$, as the $F_s(w) > F_c(w)$, $\forall w$ and $\partial V(w, a)/\partial w > 0$.

7.2 Proof of Proposition 2

There exists unique $a^*(w) \in [0, 1]$, given $w > 0$.

$V_s(w, a) = u(w) + \beta \mathbf{E}_{s,a'} V(w'_s, a')$ is independent of a. Thus, we can draw V_s with horizontal line onto the ability-value space.

$V_c(w, a) = \max_\rho \{u(w - e - \rho) + \beta \pi(a, \rho) \mathbf{E}_{c,a'} V(w'_c, a') + \beta(1 - \pi(a, \rho)) \mathbf{E}_{s,a'} V(w'_s, a')\}$.

Claim for $\forall w > 0$ and $\forall a_1 < a_2$, $V_c(w, a_1) < V_c(w, a_2)$. Then we can conclude that $V_c(w, a)$ is increasing with respect to w. This proves the proposition, as these two curves either intersect as a unique a_i^* or do not intersect at all, in the latter case $a^* = 5$.

$$\begin{aligned}
V_s(w, a_2) &= u(w - e - \rho_2) + \beta \pi(a_2, \rho_2) \mathbf{E}_{c,a'} V(w'_c, a') + \beta(1 - \pi(a_2, \rho_2)) \mathbf{E}_{s,a'} V(w'_s, a') \\
&\geq u(w - e - \rho_1) + \beta \pi(a_2, \rho_1) \mathbf{E}_{c,a'} V(w'_c, a') + \beta(1 - \pi(a_2, \rho_1)) \mathbf{E}_{s,a'} V(w'_s, a') \\
&\geq u(w - e - \rho_1) + \beta \pi(a_1, \rho_1) \mathbf{E}_{c,a'} V(w'_c, a') + \beta(1 - \pi(a_1, \rho_1)) \mathbf{E}_{s,a'} V(w'_s, a')
\end{aligned}$$

The last inequality holds since $\partial \pi(a, \rho)/\partial a > 0$.

7.3 Proof of Proposition 3

Suppose $\partial^2 \pi(a, \rho)/\partial \rho \partial a < 0$ and $\partial^2 \pi(a, \rho)/\partial \rho^2$. Then, $\rho_1 \geq \rho_2$, where ρ_1 and ρ_2 are the V_c maximizing level of private tutoring when the ability is a_1 and a_2, respectively.

The first order condition of maximizing V_c implies

$$
\begin{aligned}
u'(w - e - \rho_1) &= \beta\pi_\rho(a_1, \rho_1)(\mathbf{E}_{c,a'}V(w'_c, a') - \mathbf{E}_{s,a'}V(w'_s, a')) \\
u'(w - e - \rho_2) &= \beta\pi_\rho(a_2, \rho_2)(\mathbf{E}_{c,a'}V(w'_c, a') - \mathbf{E}_{s,a'}V(w'_s, a'))
\end{aligned}
$$

where ρ_1 and ρ_2 are the maximizing arguments and π_ρ is the partial derivative of π with respect to ρ.

Suppose $\rho_1 < \rho_2$, then $u'(w - e - \rho_1) < u'(w - e - \rho_2)$ by the concavity of utility function. By FOC above, it holds that $\pi_\rho(a_1, \rho_1) < \pi_\rho(a_2, \rho_2)$. But it leads to contradiction, as $\pi_\rho(a_1, \rho_1) > \pi(a_1, \rho_2) > \pi(a_2, \rho_2)$ as $\partial^2\tau(a, \rho)/\partial\rho\partial a < 0$, $\partial^2\tau(a, \rho)/\partial\rho^2$, $a_1 < a_2$ and $\rho_1 < \rho_2$. Thus $\rho_1 \geq \rho_2$.

7.4 Proof of Proposition 4

Let $\partial^2\pi(a, \rho)/\partial\rho^2$ and $w_1 < w_2$. Then for a fixed a, $\rho_1 \leq \rho_2$, where ρ_1 and ρ_2 are the V_c maximizing level of private tutoring when the wage is w_1 and w_2, respectively.

From the first order condition of maximizing V_c,

$$
\begin{aligned}
u'(w_1 - e - \rho_1) &= \beta\pi_\rho(a, \rho_1)(\mathbf{E}_{c,a'}V(w'_c, a') - \mathbf{E}_{s,a'}V(w'_s, a')) \\
u'(w_2 - e - \rho_2) &= \beta\pi_\rho(a, \rho_2)(\mathbf{E}_{c,a'}V(w'_c, a') - \mathbf{E}_{s,a'}V(w'_s, a'))
\end{aligned}
$$

Suppose $\rho_1 > \rho_2$, then $w_1 - \rho_1 - e < w_2 - \rho_2 - e$ and by concavity of utility function,

$$
\frac{u'(w_1 - e - \rho_1)}{u'(w_2 - e - \rho_2)} = \frac{\pi_\rho(a, \rho_1)}{\pi_\rho(a, \rho_2)} > 1.
$$

However, this leads to a contradiction as $\pi_\rho(a, \rho_1) < \pi_\rho(a, \rho_2)$ from $\rho_1 > \rho_2$.